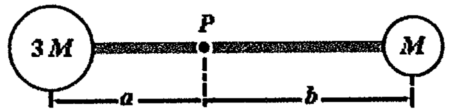
EXAM #3: SPRING SEMESTER 2017 –PHY2048
27 April 2017 – Instructor: Enrique del Barco

NAME SOLUTIONS

P.1 (25 points): The rigid body shown is rotated about an axis perpendicular to the paper and through the point P. If $M = 0.40$ kg, $a = 30$ cm, and $b = 50$ cm, how much work is required to take the body from rest to an angular speed of 5.0 rad/s? Neglect the mass of the connecting rods and treat the masses as particles.



Work-energy theorem:

$$\Delta K = W \qquad \Delta K = K_f - K_i \underset{\uparrow}{=} K_f$$

($K_i = 0$ at rest)

Rotational kinetic energy:

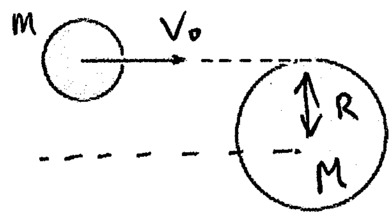
$$K_R = \frac{1}{2} I_{TOTAL} \omega^2 = \frac{1}{2}\left(I_{3M} + I_M\right)\omega^2$$

All reduces to calculate the moments of inertia of the two particles: $I = mr^2$

$$K_F = \frac{1}{2}\left[3Ma^2 + Mb^2\right]\omega^2 = \frac{1}{2}\left[3 \times 0.4\,kg\,(0.3\,m)^2 + 0.4\,kg\,(0.5\,m)^2\right]5^2\,\frac{rad}{s}$$

$$\boxed{W = K_F = 2.6\ J}$$

P.2 (25 points): A particle of mass $m = 0.10$ kg and speed $v_0 = 5.0$ m/s collides and sticks to the end of a uniform solid cylinder of mass $M = 1.0$ kg and radius $R = 20$ cm. If the cylinder is initially at rest and is pivoted about a frictionless axle through its center, what is the final angular velocity (in rad/s) of the system after the collision?



Conservation of angular momentum

$$L_i = L_f$$

Initial angular momentum:

$$L_i = \vec{r} \times \vec{p} = R m V_0$$

Final angular momentum:

$$L_f = I_{TOTAL}\,\omega = \left(mR^2 + \tfrac{1}{2}MR^2\right)\omega$$

$$\left\{ I_{TOTAL} = I_m + I_{cylinder} = mR^2 + \tfrac{1}{2}MR^2 \right\}$$

Making the angular momenta equal:

$$R m V_0 = \left(mR^2 + \tfrac{1}{2}MR^2\right)\omega$$

and solving for ω:

$$\boxed{\omega = \frac{R m V_0}{\left(mR^2 + \frac{1}{2}MR^2\right)} = 8.3 \text{ rad/s}}$$

P.3 (25 points): How long must a pendulum be on the Moon, where $g = 1.6$ N/kg, to have a period of 2.0 s?

this is really easy!

$$T = \frac{1}{f} \quad \left(f = \frac{\omega}{2\pi} \right) \quad T = \frac{2\pi}{\omega}$$

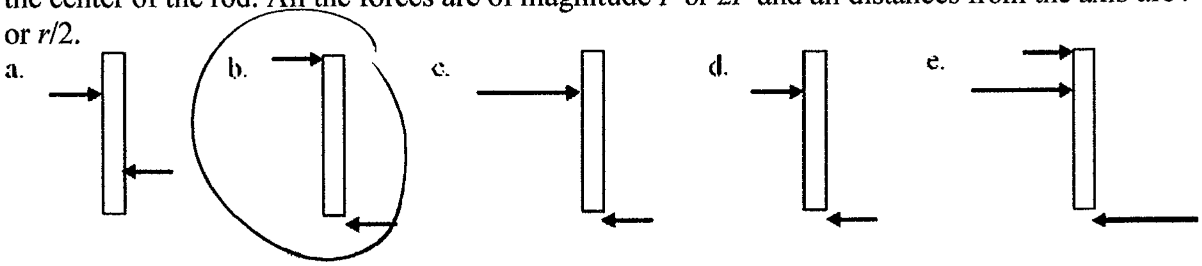
$$\omega = \sqrt{\frac{g}{L}} \rightarrow T = 2\pi\sqrt{\frac{L}{g}} \rightarrow \left(\frac{T}{2\pi}\right)^2 = \frac{L}{g} \rightarrow L = g\left(\frac{T}{2\pi}\right)^2$$

$$\boxed{L = 1.6\,\frac{N}{Kg}\left(\frac{2s}{2\pi}\right)^2 = 0.16\,m}$$

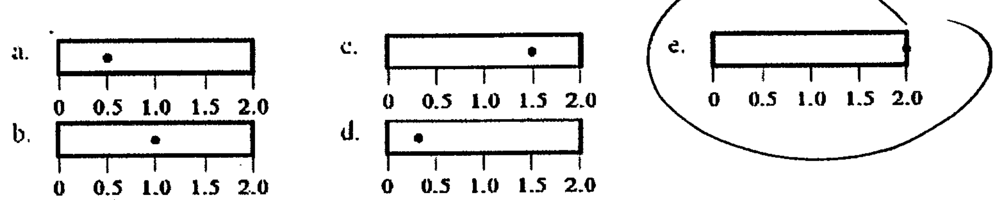
Q1 (5 points) The angular speed of the hour hand of a clock, in rad/min, is

a. $\frac{1}{1\,800}\pi$.
b. $\frac{1}{60}\pi$.
c. $\frac{1}{30}\pi$.
d. π.
e. 120π.

Q2 (5 points) Which of the following diagrams shows the greatest magnitude net torque with a zero net force? All the rods, of length 2r, rotate about an axis that is perpendicular to the rod and fixed in the center of the rod. All the forces are of magnitude F or $2F$ and all distances from the axis are r or $r/2$.



Q3 (5 points) The diagram below shows five 20-kg rods of the same 2.0-m length free to rotate about axes through the rods, as indicated. Which rod experiences the greatest ~~magnitude~~ gravitational torque?



Q4 (5 points) A solid sphere, spherical shell, solid cylinder and a cylindrical shell all have the same mass m and radius R. If they are all released from rest at the same elevation and roll without slipping, which reaches the bottom of an inclined plane first?

a. solid sphere
b. spherical shell
c. solid cylinder
d. cylindrical shell
e. all take the same time

Q5 (5 points) What makes the periodic motion of a mass attached to a string to change its frequency?

a. Increasing the time during which the system oscillates
b. Hanging the system from the ceiling
c. Changing the length of the spring without changing its spring constant
d. Friction
e. Larger gravitational acceleration