OSCILLATIONS ABOUT EQUILIBRIUM

Some definitions for periodic motion:

Period: $T \equiv$ time required for one cycle of a periodic motion

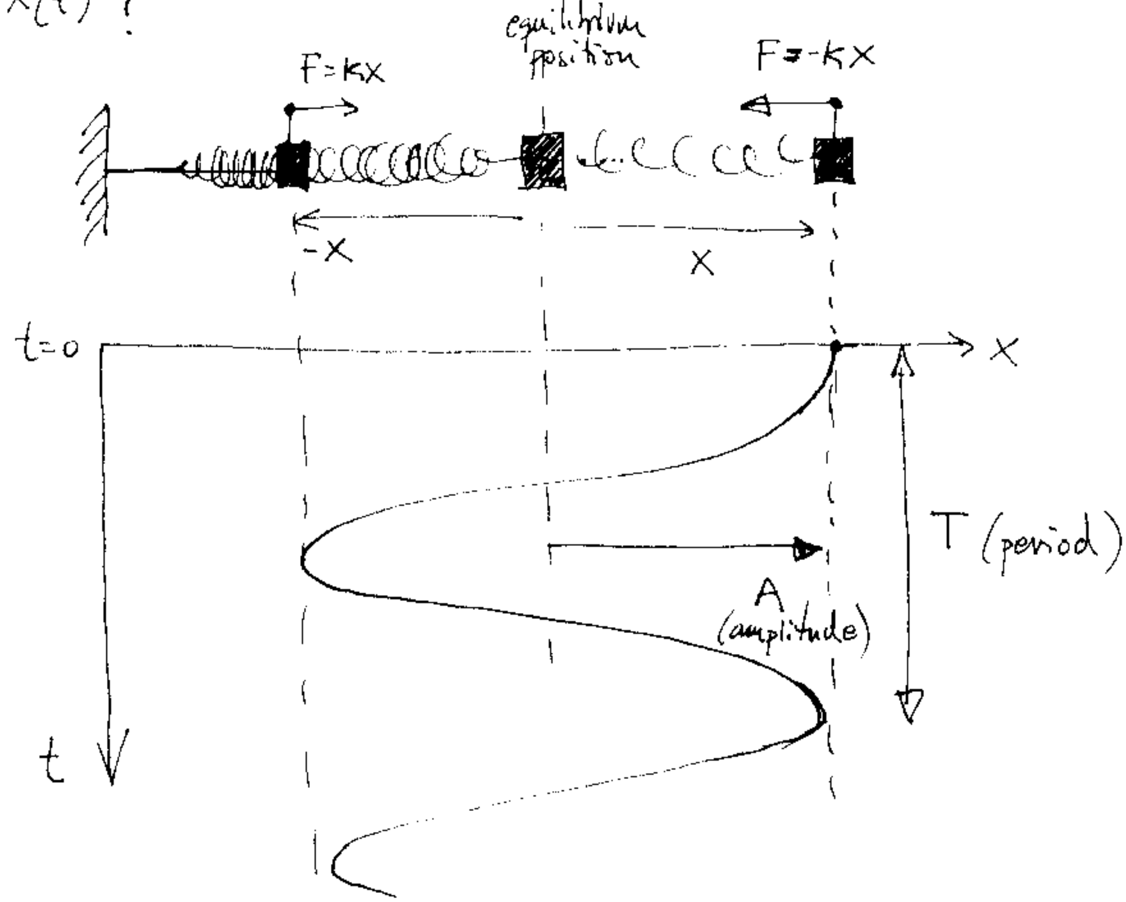
Frequency: $f = \frac{1}{T}$ $\quad [f] = \frac{1}{s} = Hz \quad 1\,Hz = 1$ cycle/second

Circular periodic motion:

Frequency (angular): $\omega = 2\pi f \quad [\omega] \equiv \frac{rad}{second} = \frac{1}{s} = Hz/2\pi$

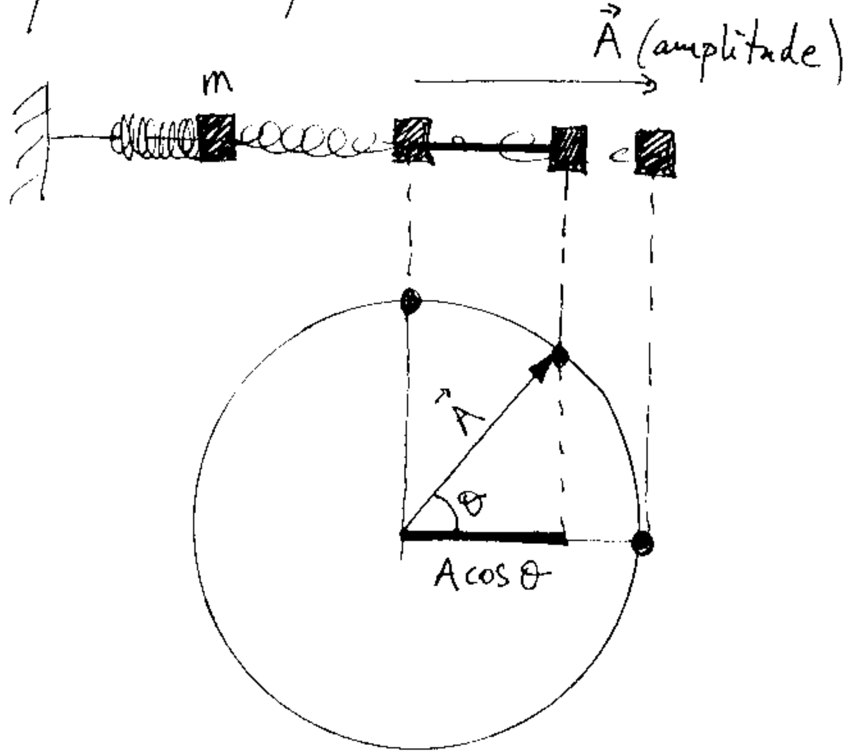
How to describe the periodic motion of an object?

$x(t)$?



What's the expression of "x" as a function of time ?
→ We need calculus to find it.

Hint: We can use what we know about circular motion to find the expression.



this means that the position of the mass "m" is given by

$$X = A\cos\theta$$

We know that in ~~rotational~~ motion, $\theta = \omega t$, so:

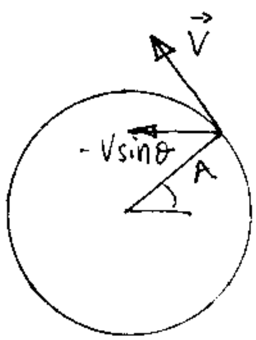
$$\boxed{X = A\cos(\omega t) = A\cos(2\pi f t) = A\cos\left(\frac{2\pi}{T}t\right)}$$

* 1st. equation of periodic motion for position.

$X_{MAX} = \pm A \quad \left[\cos(\omega t) = \pm 1 \Rightarrow \omega t = n\pi \Rightarrow t = \frac{n\pi}{\omega} = \frac{n\pi}{2\pi}T = \frac{n}{2}T\right]$

$X_{MIN} = 0 \quad \left[\cos(\omega t) = 0 \Rightarrow \omega t = (2n+1)\frac{\pi}{2} \Rightarrow t = n\frac{T}{4}\right]$

What about the velocity?



$V_x = -V\sin\theta = -A\omega\sin(\omega t)$

$V = r\omega$

$$\boxed{V = -A\omega\sin(\omega t)}$$

2nd equation of periodic motion for velocity.

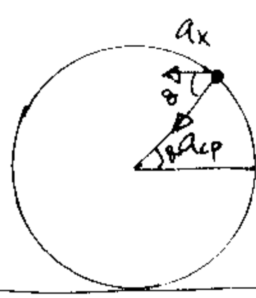
$V_{MAX} = -A\omega \quad [\sin(\omega t) = \pm 1]$

V_{MAX}: $[\sin(\omega t) = \pm 1 \Rightarrow \omega t = (2n+1)\frac{\pi}{2} \Rightarrow t = \frac{(2n+1)}{\omega}\frac{\pi}{2}$

$\Rightarrow t = (2n+1)\frac{\cancel{\pi}}{2}\frac{T}{2\cancel{\pi}} = (2n+1)\frac{T}{4}$

$V_{MIN} = 0 \quad [\sin(\omega t) = 0 \Rightarrow \omega t = n\pi \Rightarrow t = n\frac{T}{2}]$

What about the acceleration?



$a_{cp} = r\omega^2$

$a_x = -a_{cp}\cos\theta = -r\omega^2\cos(\omega t)$

$$\boxed{a = -A\omega^2\cos(\omega t)}$$

$a_{MAX} = -A\omega^2 \quad [\cos(\omega t) = \pm 1,\ t = n\frac{T}{2}]$

$a_{MIN} = 0 \quad [\cos(\omega t) = 0,\ t = n\frac{T}{4}]$

Summary of equations of periodic motion

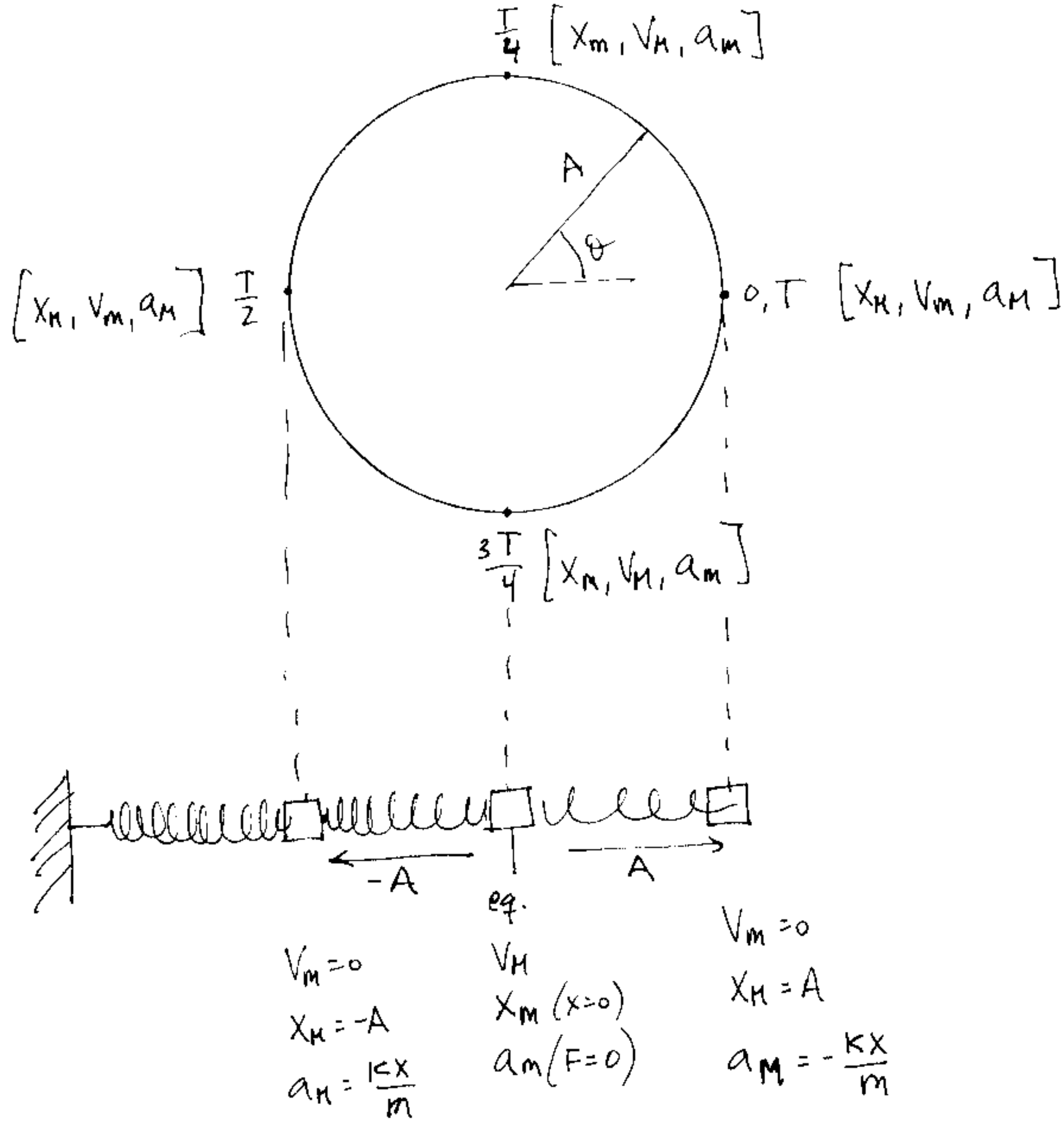
$$x = A\cos(\omega t) = A\cos(2\pi f t) = A\cos\left(\frac{2\pi}{T}t\right)$$

$$v = -A\omega\sin(\omega t)$$

$$a = -A\omega^2\cos(\omega t)$$

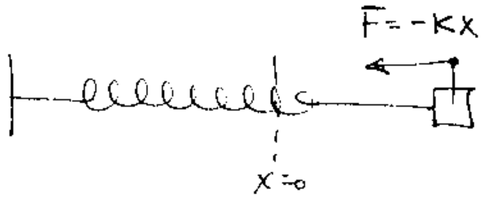
$x_M = \pm A \rightarrow t = n\frac{T}{2}$ $\quad$ $v_M = \pm A\omega \rightarrow t = (2n+1)\frac{T}{4}$ $\quad$ $a_M = \pm A\omega^2 \rightarrow t = n\frac{T}{2}$

$x_m = 0 \rightarrow t = (2n+1)\frac{T}{4}$ $\quad$ $v_m = 0 \rightarrow t = n\frac{T}{2}$ $\quad$ $a_m = 0 \rightarrow t = (2n+1)\frac{T}{4}$



HOW TO FIND THE PERIOD IN SEVERAL SITUATIONS ?

(A) THE PERIOD OF A MASS ON A SPRING



$F = -kx = ma$

we have seen that $a = -A\omega^2 \cos(\omega t)$, we substitute this in the previous formula and we obtain;

$$m\left[-A\omega^2 \cos(\omega t)\right] = -kx$$

we know $x = A\cos(\omega t)$, then;

$$m\left[\cancel{-}A\omega^2 \cancel{\cos(\omega t)}\right] = \cancel{-}k\left[A\cancel{\cos(\omega t)}\right]$$

we obtain

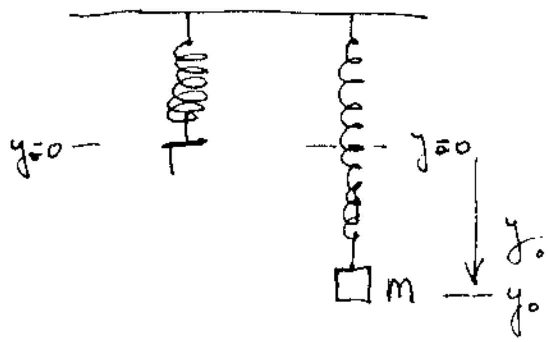
$$m\cancel{A}\omega^2 = k\cancel{A} \rightarrow \omega^2 = \frac{k}{m} \rightarrow \boxed{\omega = \sqrt{\frac{k}{m}}}$$

Since $T = 2\pi\omega$ then,

$$\boxed{T = 2\pi\sqrt{\frac{k}{m}}}$$

Increasing m or decreasing k the period will decrease

(A1) PARTICULAR CASE = VERTICAL SPRING



$\rightarrow \quad mg = ky_0$

so $\boxed{y_0 = \frac{m}{k} g}$

New equilibrium position

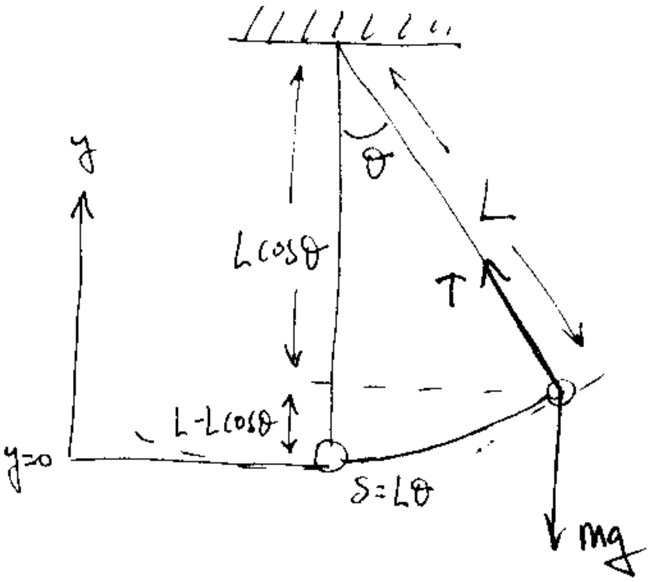
We know that $T = 2\pi \sqrt{\frac{k}{m}} \rightarrow \frac{k}{m} = \left(\frac{T}{2\pi}\right)^2 \rightarrow \frac{m}{k} = \left(\frac{2\pi}{T}\right)^2$

then the new eq. position is related T through:

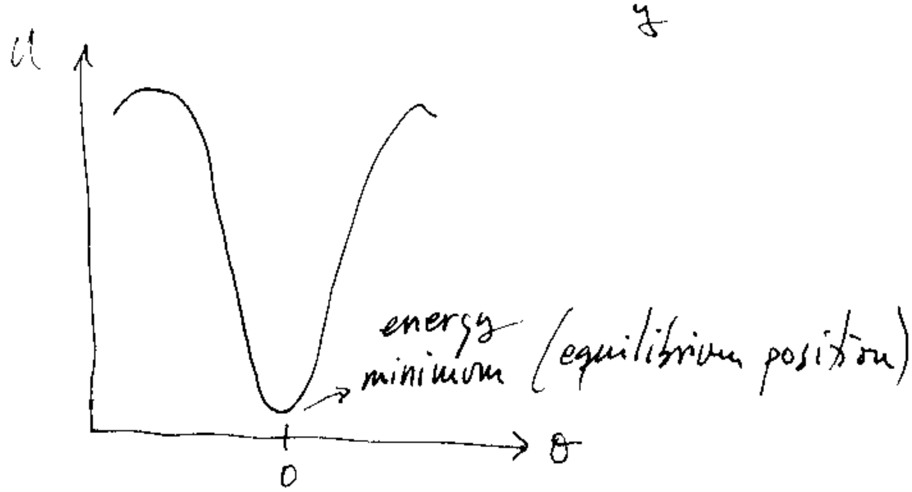
$$\boxed{y_0 = \frac{m}{k} g = \left(\frac{2\pi}{T}\right)^2 g}$$

But the period of oscillation will be the same.

(8) THE PERIOD OF A PENDULUM



Potential energy: $U = mg\underbrace{(L - L\cos\theta)}_{y} = mgL(1-\cos\theta)$



Tangential force always points to the equilibrium position.

$$F = mg\sin\theta$$

The arc covered ~~on~~ θ, is $s = L\theta$

For small angles, we can aproximate $\sin\theta \approx \theta$. Then:

$$F = mg\theta$$

But we know, $\theta = S/L$, so;

$$F = mgS/L = \left(\frac{mg}{L}\right)S$$

This eq. has the same form than the spring force;

$$F = KX$$

So the equivalence gives us the spring constant of a pendulum which should be called, the restoring force constant:

$$K = \frac{mg}{L} \rightarrow \frac{M}{K} = \frac{L}{g}$$

finally, the period of a pendulum (for small angle oscillations) is:

$$\boxed{T = 2\pi\sqrt{\frac{L}{g}}}$$

if g increases T decreases (faster oscillation in Earth than Moon)

if L increases T increases (~~faster~~ faster oscill. for shorter pend.)