NEWTON'S LAWS OF MOTION

$\boxed{\text{First Law}}$ An object moving with constant velocity will move at the same velocity forever if no net forces act on it.

$\boxed{\text{second law}}$ An object of mass (m) has an acceleration ($\vec{a}$) equal to the net force vector ($\sum \vec{F}$) divided by the mass.

$$\boxed{\vec{a} = \frac{\sum \vec{F}}{m} \quad \text{or} \quad \sum \vec{F} = m\vec{a}}$$

$[F] \equiv N = kg\,\frac{m}{s^2}$
units

where ; $\sum_{i=1}^{N} \vec{F}_i = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \cdots + \vec{F}_N$

Remember how to add (sum) vectors

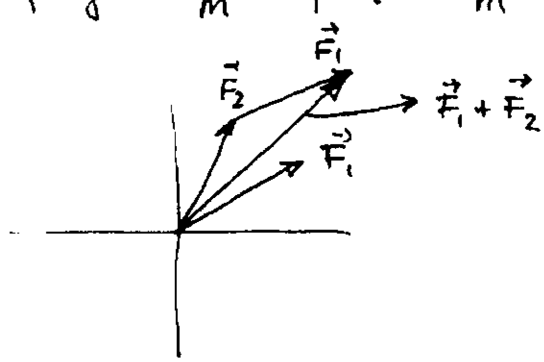
i) by components ;

$$\sum \vec{F} = \sum F_x\,\hat{x} + \sum F_y\,\hat{y} + \sum F_z\,\hat{z} \quad \left[\sum_{i=1}^{N} F_{x_i} = F_{x_1} + F_{x_2} + \cdots\right]$$

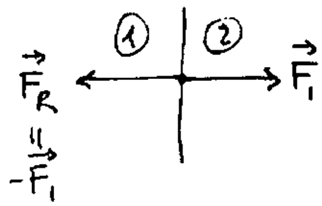
so, in the same way we can study the acceleration by its components :

$$a_x = \frac{\sum F_x}{m}\ ,\ a_y = \frac{\sum F_y}{m}\ ,\ a_z = \frac{\sum F_z}{m}$$

ii) geometrically :



$\boxed{\text{third law}}$ For every force that acts on an object there is a reaction force that is equal in magnitude and opposite in direction.

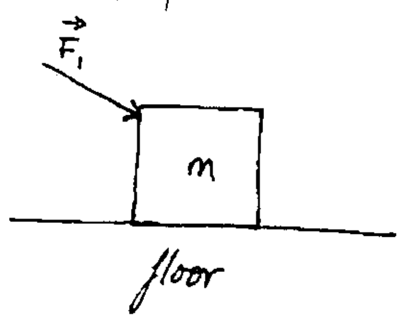


FREE-BODY DIAGRAMS

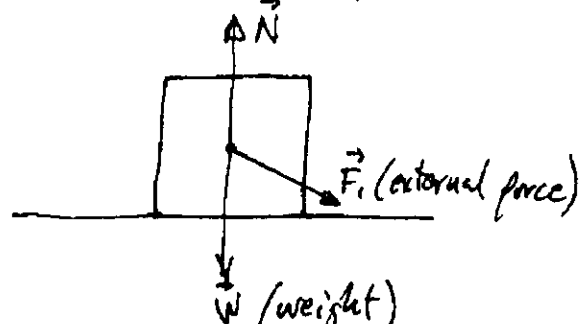
For non-rotational motion, all the forces acting on an object can be taken to act on a point. This point usually corresponds to the center of mass of the object.

i) First step: Figure out each and every external force acting on an object, taking into account direction and magnitude.

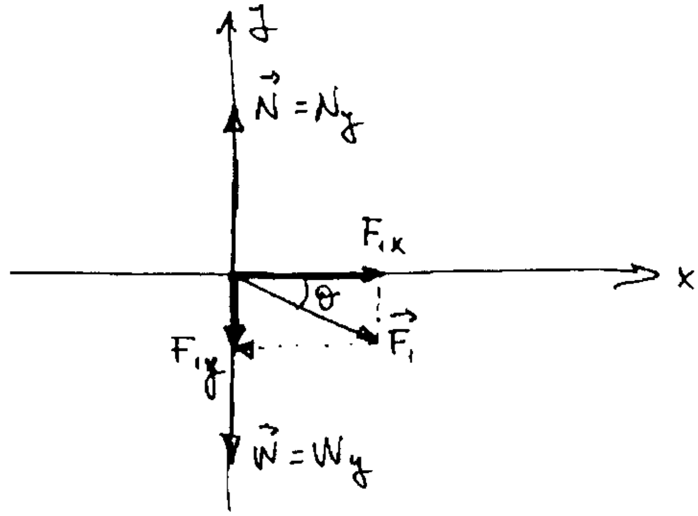
For example:



Forces acting on the object:



ii) <u>Second step</u>: Project each force on the coordinate system axes. For this, an appropriate coordinate system should be chosen.



iii) <u>third step</u>: Work out the resulting net force on each of the axes (x, y).

on the x-axis: $\sum F_x = F_{1x}$

on the y-axis: $\sum F_y = N_y + F_{1y} - W_y$

where: $F_x = F_1 \cos\theta$

$F_y = F_1 \sin\theta$

iv) <u>Fourth step</u>: Apply the equations of motion seen in the previous chapter by using the acceleration generated by the net forces.

$$a_x = \sum \frac{F_x}{m} = F_{1x}/m$$

$$a_y = \sum \frac{F_y}{m} = (N_y - F_{1y} - W_y)/m$$

Consequently, the equations of motion are;

$$\left.\begin{aligned} x &= x_0 + v_{0x}t + \frac{1}{2}a_x t^2 \\ y &= y_0 + v_{0y}t + \frac{1}{2}a_y t^2 \end{aligned}\right\}$$

$$\boxed{\begin{aligned} x &= x_0 + v_{0x}t + \frac{1}{2m}\sum F_x t^2 \\ y &= y_0 + v_{0y}t + \frac{1}{2m}\sum F_y t^2 \end{aligned}}$$

And for the velocity:

$$\left.\begin{aligned} v_x &= v_{0x} + a_x t \\ v_y &= v_{0y} + a_y t \end{aligned}\right\}$$

$$\boxed{\begin{aligned} v_x &= v_{0x} + \frac{\sum F_x}{m} t \\ v_y &= v_{0y} + \frac{\sum F_y}{m} t \end{aligned}}$$

and finally (the second Newton's law):

$$\boxed{\begin{aligned} a_x &= \frac{\sum F_x}{m} \\ a_y &= \frac{\sum F_y}{m} \end{aligned}}$$

APPLICATIONS OF NEWTON'S LAWS

MASS AND WEIGHT

The weight of an object, according to the 2nd Newton's law, is the force exerted on the object of mass "m" by the gravitational attraction to the Earth.

$$\vec{F}_g = m\vec{g}$$

The weight is the magnitude of this force:

$$F_g \equiv W = mg$$

And the force is always directed vertically downwards.

WEIGHT AND APPARENT WEIGHT

weight ($\vec{W}$) and apparent weight ($\vec{W}_a$)

Our weight is a force that results from the action of gravity on our mass:

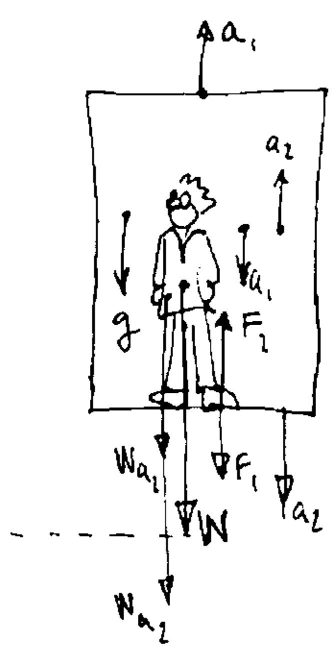
$$\boxed{\vec{W} = m\vec{g}}$$

where $\vec{g}$ is the gravity acceleration.

Apparent weight (W_a):

$$\vec{W}_a = m\sum \vec{a}$$

If we are in an elevator, there is one acceleration exerted to us by the floor of the elevator that should be add to the acceleration of gravity.



case a_1: a_1 is opposite to g

$$W_a = m(g + a_1) > W = mg$$

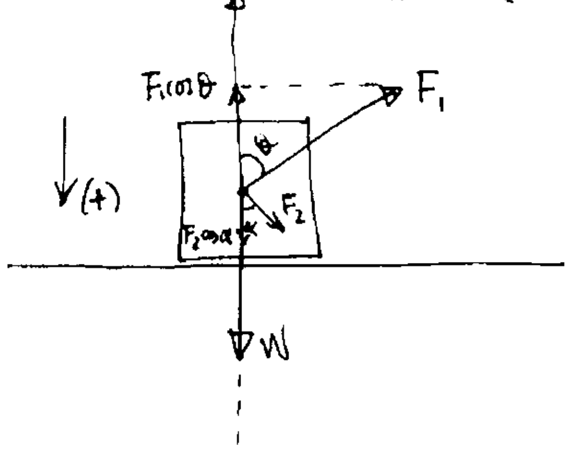
case a_2: a_1 is in g-direction

$$W_a = m(g - a_1) < W = mg$$

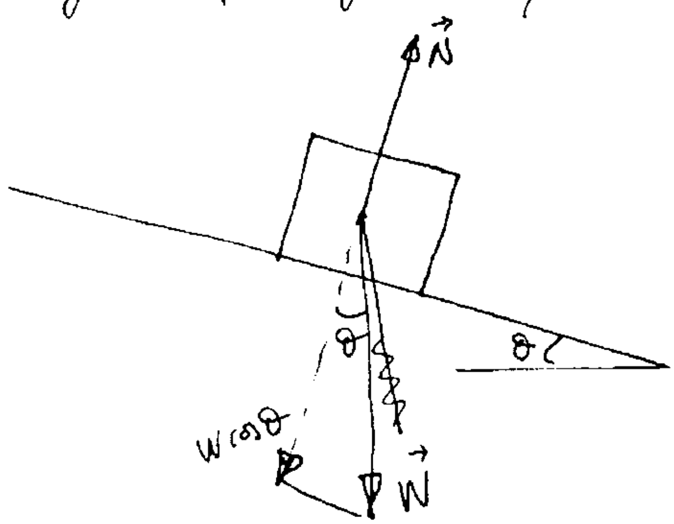
NORMAL FORCES

The normal force is the reaction force generated by and object to equal the sum of all forces perpendicular to the object.

$$N = -\sum F_{\perp} = -(-F_1 \cos\theta + F_2 \cos\alpha + W)$$



This is alway true, even if the surface is tilted:

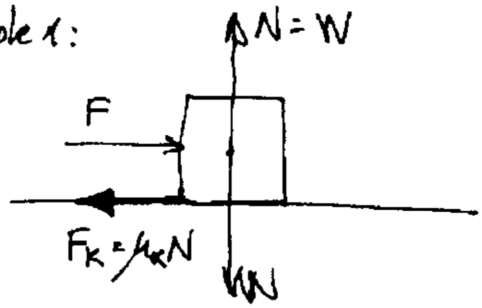


$$\boxed{N = -W\cos\theta}$$

KINEMATIC FRICTION AND STATIC FRICTION

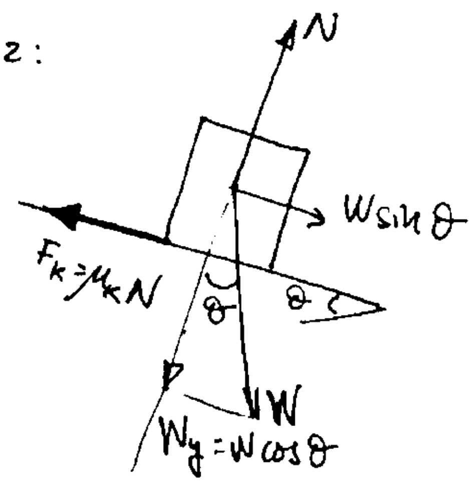
$\vec{F}_K = \mu_K \vec{N}$

Example 1:



$F_K = \mu_K N = \mu_K W$

Example 2:



$F_K = \mu_K N = \mu_K W\cos\theta$

STATIC FRICTION

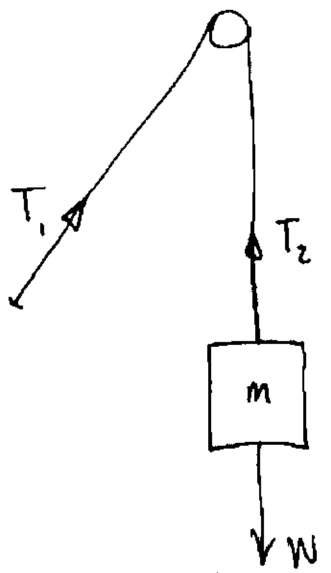
$0 \le F_s \le F_{s,max} \longrightarrow F_{s,max} = \mu_s N$

STRINGS

We only consider strings with $m=0$

$\Downarrow$

" The tension is the same along the whole string "



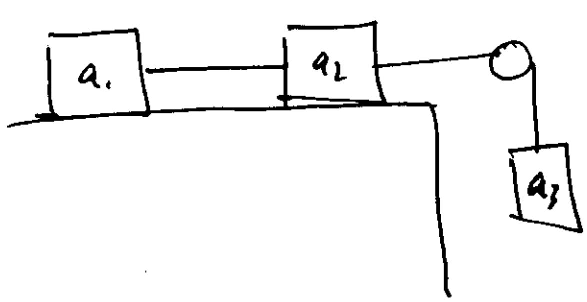
if $m_{string}=0 \Rightarrow T_1=T_2$

If the box is not moving, then $T=T_1=T_2=W$

this comes from; $\sum\vec{F}=m\vec{a}$

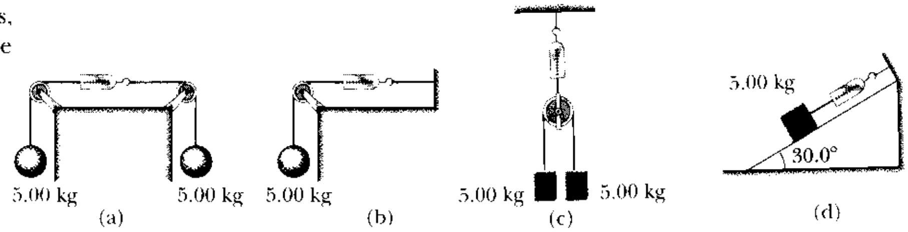
$$+W-T_2=0 \Rightarrow W=T_2=T$$

* <u>connected objects:</u>

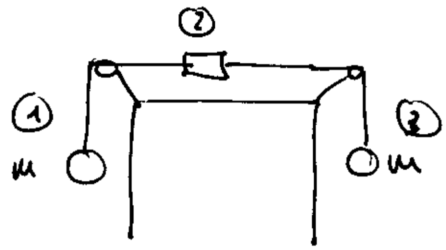


$a_1=a_2=a_3=a$

The systems shown in Figure P5.23 are in equilibrium. If the spring scales are calibrated in newtons, what do they read? Ignore the masses of the pulleys and strings, and assume the pulleys and the incline in part (d) are frictionless.

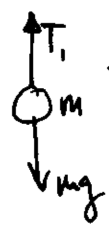


a)



1) isolate masses.

1



$T_1 - mg = ma = 0$

$T_1 = mg$

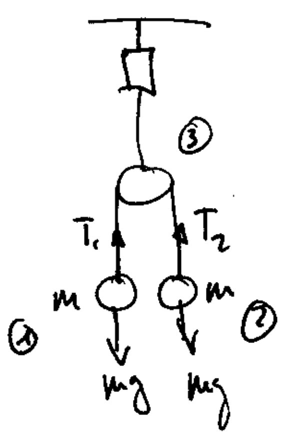
2

$$\boxed{T_1 = mg = 5\text{kg}\, 9.81\frac{m}{s^2} = 49N}$$

b) The same thing.

First :

c)



1 $T_1 - mg = ma_1$

2 $T_2 - mg = ma_2$

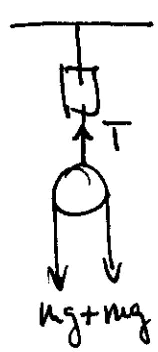
3 $T_1 = T_2$

4 $a_1 = a_2$

$T - mg = ma = 0$

$$\boxed{T = mg}$$

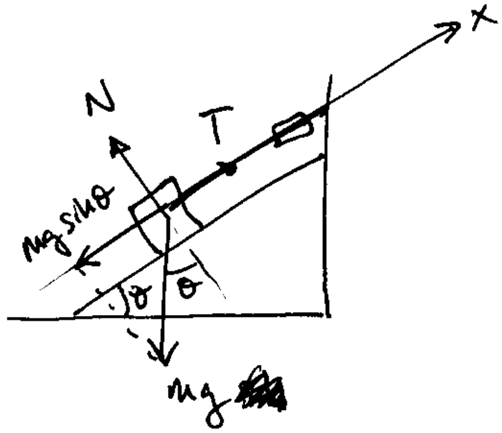
second



$T - 2mg = \cancel{(2m)a}_0 \rightarrow T = 2mg$

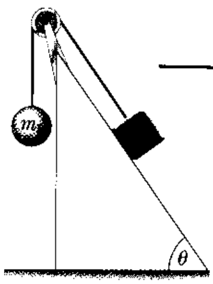
$$\boxed{T = 98N}$$

d)

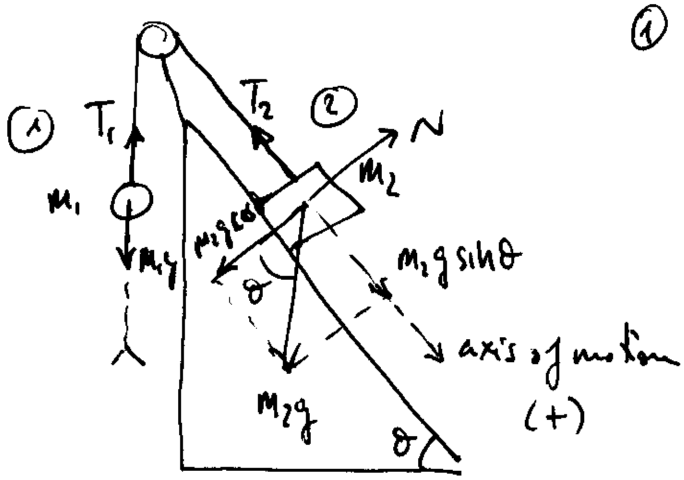


$$x)\quad T - mg\sin\theta = 0 \rightarrow T = mg\sin\theta$$

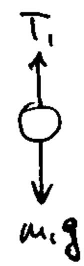
$$T = 5\,kg\, 9.81\,\frac{m}{s^2} \cdot \sin 30^\circ = 24.5\,N$$



Two objects are connected by a light string that passes over a frictionless pulley as shown in Figure P5.28. Draw free-body diagrams of both objects. Assuming the incline is frictionless, m_1 = 2.00 kg, m_2 = 6.00 kg, and θ = 55.0°, find (a) the accelerations of the objects, (b) the tension in the string, and (c) the speed of each object 2.00 s after they are released from rest.

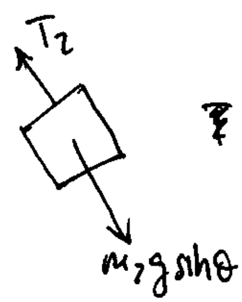


1



$$T_1 - m_1 g = m_1 a_1$$

2



$$m_2 g \sin\theta - T_2 = m_2 a_2$$

connected objects:

$a_1 = a_2$

massless string

$m_{st} = 0$

$T_1 = T_2$

$$T - m_1 g = m_1 a$$

$$m_2 g \sin\theta - T = m_2 a$$

$$a = \frac{T - m_1 g}{m_1} = \frac{m_2 g \sin\theta - T}{m_2}$$

$$m_2 T - m_1 m_2 g = m_1 m_2 g \sin\theta - m_1 T$$

$$\boxed{T = \frac{m_1 m_2 g(\sin\theta + 1)}{m_1 + m_2} = 26.7\,N}$$

a) a?

b) T?

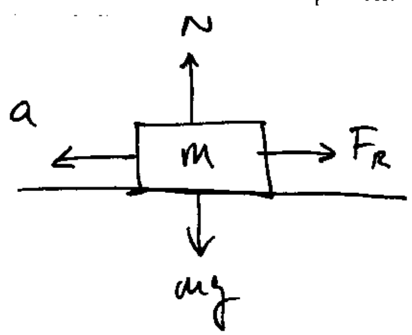
c) $v(t = 2s)$?

b) $a = \dfrac{T - m_1 g}{m_1} = 3.57 \, \dfrac{m}{s^2}$

c) $V(t=2s) = \cancel{V_0} + at = 3.57 \, \dfrac{m}{s^2} \cdot 2s = 7.14 \, \dfrac{m}{s}$

$\cancel{V_0}$: 0

Your 3.80-kg physics book is next to you on the horizontal seat of your car. The coefficient of static friction between the book and the seat is 0.650, and the coefficient of kinetic friction is 0.550. Suppose you are traveling at 72.0 km/h = 20.0 m/s and brake to a stop over a distance of 45.0 m. (a) Will the book start to slide over the seat? (b) What force does the seat exert on the book in this process?



$m = 3.80 \, kg$ $\quad V_0 = 72 \, km/h = 20 \, m/s$

$\mu_s = 0.65$

$\mu_k = 0.55$ $\quad x_f = 45m \quad x_0 = 0$

a) will slide?

b) Force on the book.

we need to know the acceleration:

$$\underbrace{V_f^2}_{0} = \underbrace{V_0^2}_{20 m/s} + 2a(\underbrace{x_f}_{45m} - \underbrace{x_0}_{0}) \rightarrow \boxed{a = -4.44 \, \frac{m}{s^2}}$$

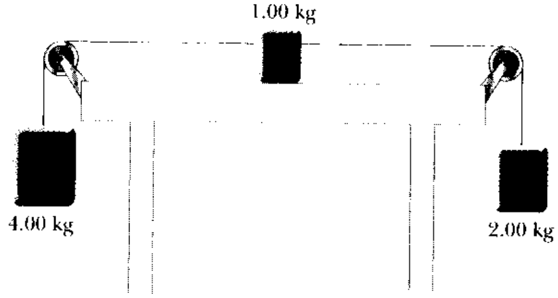
$$\Sigma F_x = ma_x \rightarrow -F_R = ma \Rightarrow \boxed{F_R = 16.9N}$$

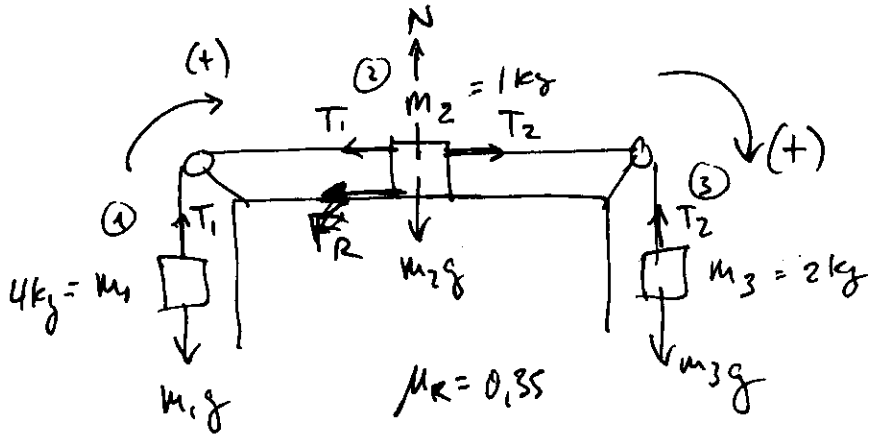
$$\Sigma F_y = ma_y \rightarrow N - mg = 0 \Rightarrow \boxed{N = mg = 37.2N}$$

$$F_s \leq \mu_s N = 0.65\,(37.2N) = \underline{24.2N} > \boxed{F_R = 16.9N}$$

The book does not slide

Three objects are connected on the table as shown in Figure P5.44. The table is rough and has a coefficient of kinetic friction of 0.350. The objects have masses of 4.00 kg, 1.00 kg, and 2.00 kg, as shown, and the pulleys are frictionless. Draw free-body diagrams of each of the objects. (a) Determine the acceleration of each object and their directions. (b) Determine the tensions in the two cords.





$a_1 = a_2 = a_3$

(1) $T_1 - m_1 g = m_1 a$

(2) $T_2 - T_1 - F_R = m_2 a$

(3) $m_3 g - T_2 = m_3 a$

$F_R = \mu_K N = \mu_K m_2 g = 0{,}35 \cdot 1kg \cdot 9.81 \frac{m}{s^2}$

$$\boxed{F_R = 3.43 N}$$

$$\left.\begin{aligned} & T_1 - m_1 g = m_1 a \\ + \quad & T_2 - T_1 - F_R = m_2 a \\ + \quad & m_3 g - T_2 = m_3 a \end{aligned}\right]$$

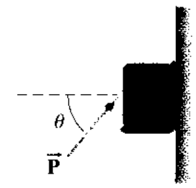
$$\cancel{T_1} - m_1 g + \cancel{T_2} - \cancel{T_1} - F_R + m_3 g - \cancel{T_2} = (m_1 + m_2 + m_3) a$$

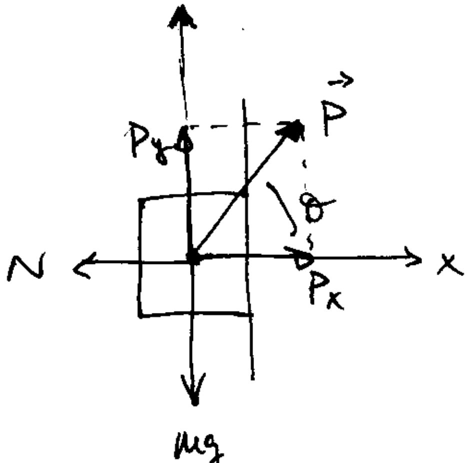
$$(m_3 - m_1) g - F_R = (m_1 + m_2 + m_3) a$$

$$\boxed{a = \frac{(m_3 - m_1) g - F_R}{m_1 + m_2 + m_3} = -2.31 \frac{m}{s^2}}$$

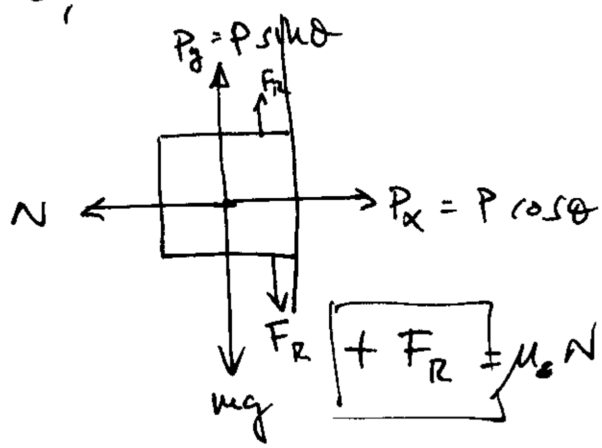
$$\boxed{\begin{aligned} T_1 &= m_1 (a + g) = 30 N \\ T_2 &= m_3 (g - a) = 24.2 N \end{aligned}}$$

● A block of mass 3.00 kg is pushed against a wall by a force $\vec{P}$ that makes a $\theta = 50.0°$ angle with the horizontal as shown in Figure P5.44. The coefficient of static friction between the block and the wall is 0.250. (a) Determine the possible values for the magnitude of $\vec{P}$ that allow the block to remain stationary. (b) Describe what happens if $|\vec{P}|$ has a larger value and what happens if it is smaller. (c) Repeat parts (a) and (b) assuming the force makes an angle of $\theta = 13.0°$ with the horizontal.





Therefore;



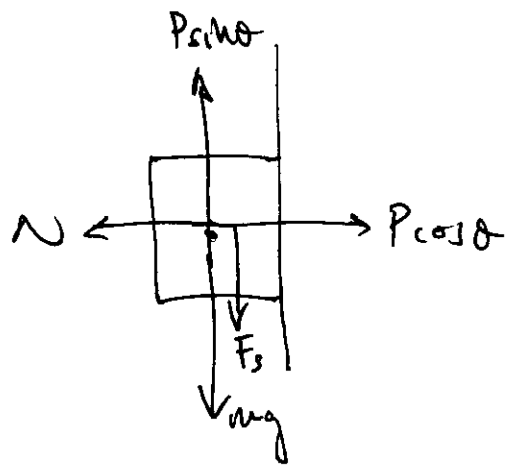
$\boxed{+F_R = \mu_s N}$

a) Maximum force:

$$F_R = \mu_s N = \mu_s P_x = \mu_s P\cos\theta$$

$$P\sin\theta - mg + \mu_s P\cos\theta = 0$$

$$\boxed{P_{MAX} = \frac{mg}{\sin\theta - \mu_s\cos\theta} = 48.6\,N}$$

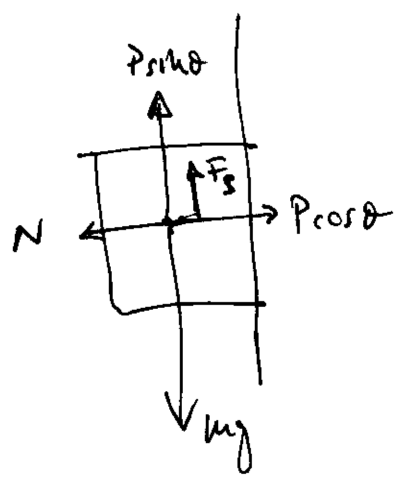


b) Minimum force:

$$F_s = \mu_s P\cos\theta$$

$$P\sin\theta + \mu_s P\cos\theta - mg = 0$$

$$\boxed{P_{min} = \frac{mg}{\sin\theta + \mu_s\cos\theta} = 31.7\,N}$$



There is a magic angle ~~for~~ below which you can not hold the box in place, no matter how strongly you push on it.

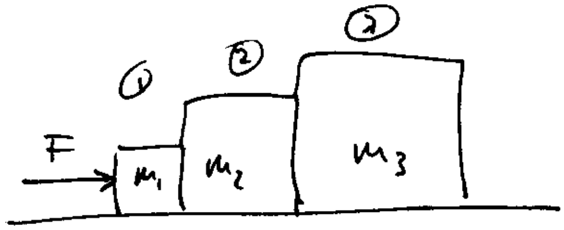
$$P_{MAX} = \infty \Rightarrow \text{when} \quad \sin\theta - \mu_s \cos\theta = 0$$

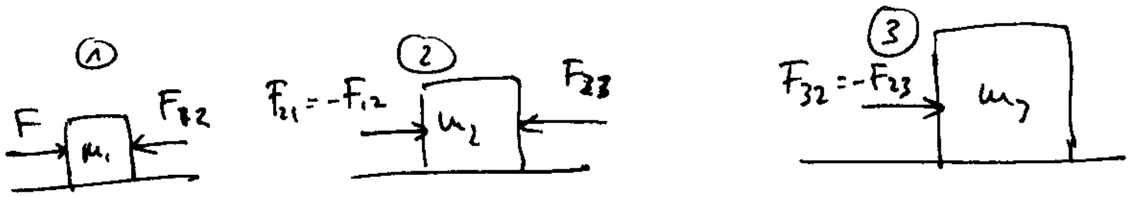
$$\sin\theta = \mu_s \cos\theta \Rightarrow \mu_s = \tan\theta$$

$$\boxed{\theta_c = 14.03^\circ}$$

17

● Three blocks are in contact with one another on a frictionless, horizontal surface as shown in Figure P5.54. A horizontal force $\vec{F}$ is applied to m_1. Take $m_1 = 2.00$ kg, $m_2 = 3.00$ kg, $m_3 = 4.00$ kg, and $F = 18.0$ N. Draw a separate free-body diagram for each block and find (a) the acceleration of the blocks, (b) the *resultant* force on each block, and (c) the magnitudes of the contact forces between the blocks. (d) You are working on a construction project. A coworker is nailing plasterboard on one side of a light partition, and you are on the opposite side, providing "backing" by leaning against the wall with your back pushing on it. Every hammer blow makes your back sting. The supervisor helps you to put a heavy block of wood between the wall and your back. Using the situation analyzed in parts (a), (b), and (c) as a model, explain how this change works to make your job more comfortable.





$$a_1 = a_2 = a_3 = a$$

(1) $F - \underbrace{F_{12}}_{P} = m_1 a \Rightarrow F - P = m_1 a$

(2) $\underbrace{F_{21}}_{-P} - \underbrace{F_{23}}_{Q} = m_2 a \Rightarrow P - Q = m_2 a$

(3) $Q = m_3 a$

$$\begin{aligned} & F - P = m_1 a \\ + \; & P - Q = m_2 a \\ + \; & Q = m_3 a \\ \hline & F = (m_1 + m_2 + m_3) a \end{aligned}$$

$$\boxed{a = 2 \frac{m}{s^2}}$$

$$\boxed{Q = m_3 a = 8 N}$$

$$\boxed{P = m_2 a + Q = 14 N}$$

$$\boxed{F = 18 N}$$