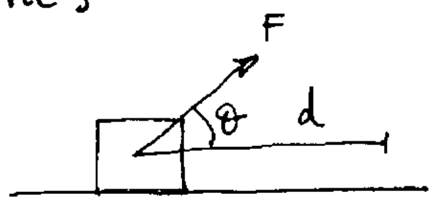
ENERGY OF A SYSTEM

7.1. WORK DONE BY A CONSTANT FORCE

$$W = \vec{F}\cdot\vec{d} = Fd\cos\theta$$
[constant force]

$$W = \int \vec{F}\,d\vec{r} = \vec{F}\int d\vec{r} = \vec{F}\vec{d}$$
[varying force : general case]



$$[W] = J = N\,m = kg\,\frac{m}{s^2}\cdot m = kg\,\frac{m^2}{s^2}$$

Note that work can be positive, negative and zero, even in the case we have displacement.

$0 \le \theta \le 90^\circ \quad W > 0$

$\theta = 90^\circ \quad W = 0$

$90^\circ \le \theta \le 180^\circ \quad W < 0$

TOTAL WORK:

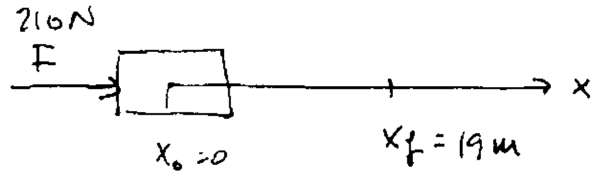
If there are several forces acting on an object that is displaced by $\vec{d}$, the total work is:

$$W_T = \sum_i W_i = \sum_i \vec{F}_i\vec{d} = \sum_i F_i\,d\cos\theta_i$$

Example

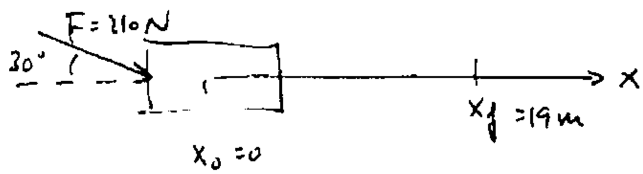
Work done by a $F = 210N$ force in moving an object 19 m when $\theta = 0$ and $\theta = 30°$

a)

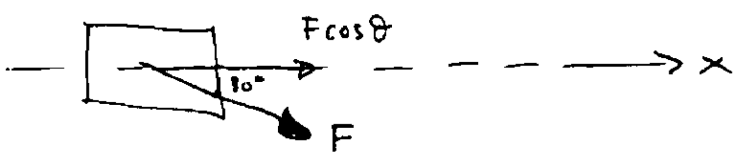


$$W = \vec{F} \cdot \vec{d} = F \cdot d \cos\theta = F \cdot d \underbrace{\cancel{\cos 0°}}_{1} = Fd = 210 N \cdot 19 m = 4 \times 10^3$$

b)

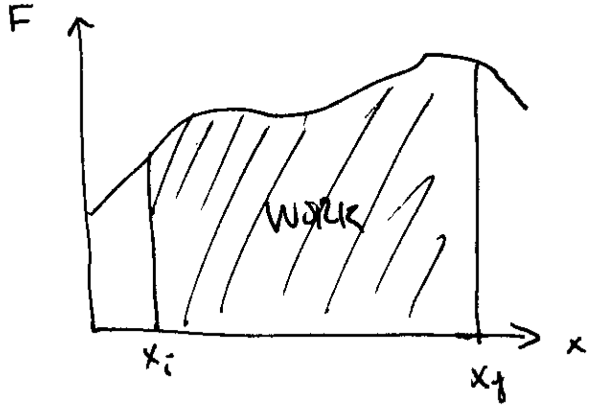


Only the projection of the force in the direction of movement will generate work.



$$W = (F \cos\theta) d = 210 N \cdot \cos 30° \cdot 19 m = \underline{3.5 \times 10^3 J}$$

WORK DONE BY A VARYING FORCE



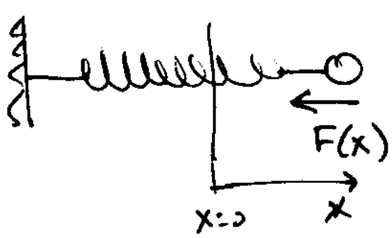
$$W = \int_{x_i}^{x_f} F\,dx$$

where $F(x)$

Total work:

$$W_N = \sum_i W_i = \int_{x_i}^{x_f} \left[\sum_i F_i(x)\right] dx$$

WORK DONE BY A SPRING



$$F_{sp} = -kx = -F_{app}$$

$$W = \int_0^x \underbrace{(+kx)}_{F_{app}}\,dx = \frac{1}{2}kx^2$$

$$\boxed{W_{sp} = \frac{1}{2}kx^2}$$

KINETIC ENERGY AND THE WORK-ENERGY THEOREM

Definition of kinetic energy

$$K = \frac{1}{2} m v^2 \qquad [K] = \text{Joule} = \text{J}$$

Definition of Work-energy theorem

$$W_{total} = \Delta K = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2 = \frac{1}{2} m \left(v_f^2 - v_i^2 \right)$$

From this theorem we know that to change the velocity of an object we need work.

* To increment the velocity we need to apply energy

$$v_f > v_i \Longrightarrow W > 0$$

* To decrease the velocity we get energy from the object

$$v_f < v_i \Longrightarrow W < 0$$

POTENTIAL ENERGY

Potential energy is the energy storaged by a system.
Potential energy is generated by conservative forces.

<u>Definition</u>: $W_c = U_i - U_f = -(U_f - U_i) = -\Delta U$

<u>Gravity potential energy</u>:

$W_c = mgy$

Applying the definition, we obtain;

$$-\Delta U = U_i - U_f = W_c = mgy$$

The change in potential energy is given by the height, y, of the object.

$$\boxed{U_g = mgy}$$, near Earth's surface

<u>Spring potential energy</u>:

$$\boxed{U_{sp} = \frac{1}{2} k x^2}$$

CONSERVATIVE FORCES AND NON-CONSERVATIVE FORCES

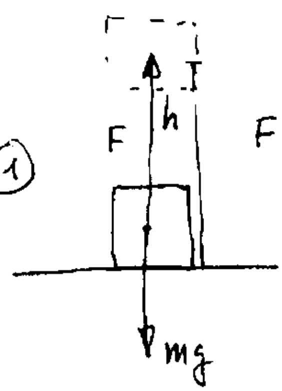
Definition:

1) A conservative force does zero work on any closed path
2) The work done by a cons. force in going from A to B is independent of the path.

Conservative forces: gravity, spring ...
Non conservative forces: Friction

Gravity:

1



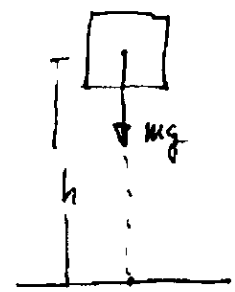
$F = mg$

$W_F = mgh$

$F_g = -mg$

$W_g = -mgh$

2



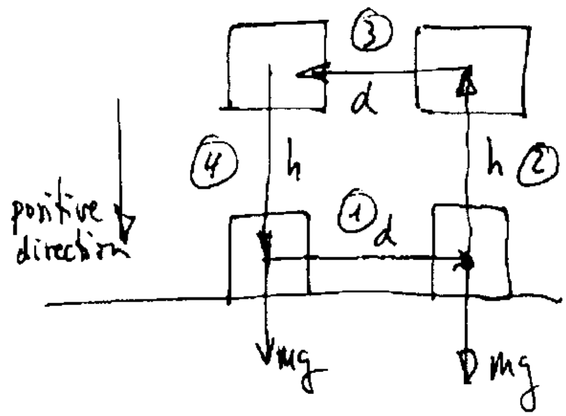
$F_g = mg$

$W_g = mgh$

Total work done by gravity $W_{Tg} = W_{g1} + W_{g2} =$

$= -mgh + mgh = 0$

Close path and conservative force (gravity)



① $W_{g1} = mg \cdot d \cos 90° = 0$

② $W_{g2} = +mgh \cos 180° = -mgh$

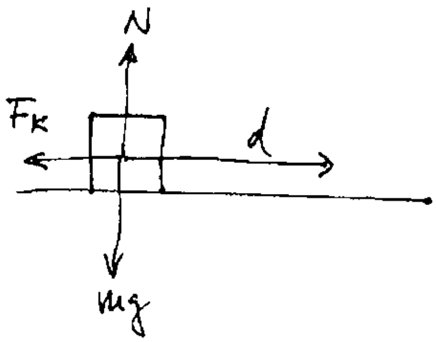
③ $W_{g3} = mgd \cos 90° = 0$

④ $W_{g4} = mgh \cos 0° = mgh$

Total work $W_{Tg} = W_{g1} + W_{g2} + W_{g3} + W_{g4} =$

$= 0 - mgh + 0 + mgh = 0$

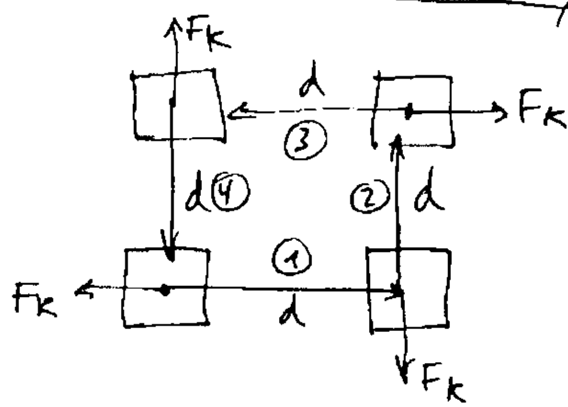
Non conservative force (friction)



$$W_K = F_K d \cos 180^\circ = -F_K d = -\mu_K mg\, d$$

After finishing pushing the box, the box remains at rest so friction force does not give back this work.

Close path with non-conservative force (friction)



$$W_{K1} = F_K d \cos 180^\circ = -F_K d$$
$$W_{K2} = F_K d \cos 180^\circ = -F_K d$$
$$W_{K3} = F_K d \cos 180^\circ = -F_K d$$
$$W_{K4} = F_K d \cos 180^\circ = -F_K d$$

Total work $W_{TK} = W_{K1} + W_{K2} + W_{K3} + W_{K4} = -4F_K d \neq 0$

CONSERVATIVE FORCES AND POTENTIAL ENERGY

$$W_c = -\Delta U = -\int F_c \, dr \qquad F_c = -\frac{dU}{dr}$$

SUMMARY :

$$W = \vec{F} \cdot \vec{d}$$

$$W_c = -\Delta U$$

$$W_T = \Delta K = W_c + W_{NC}$$

$$W_{NC} = \Delta E$$

CONSERVATION OF ENERGY

ISOLATED AND NON ISOLATED SYSTEMS

the definitions and presentation of these two topics in the book is completely arbitrarious and misinterpreted.

They talk about isolated system with friction, this makes non sense. $\Delta E = W_{NC}$, since the energy dissipated by friction goes to heat, sound, etc, so the system is not isolated.

therefore, all this chapter reduces to the next:

1) If all forces acting on a system are conservative;

$$\text{if} \quad F_{NC} = 0 \implies \Delta E = 0$$

2) If there are non-conservative forces acting ~~of~~ on the system,

$$\text{if} \quad F_{NC} \neq 0 \implies \Delta E = W_{NC}$$

8.5. POWER

$$P = \frac{\partial E}{\partial t} \qquad [P] = \frac{J}{s} = \text{Watts}$$

If this change (transfer) of energy is due to a force:

$$\boxed{P = \frac{dW}{dt} = \vec{F} \cdot \frac{d\vec{r}}{dt} = \vec{F} \cdot \vec{v}}$$

constant force

One horsepower $\boxed{1\ hp = 746\ W}$

A 0.600-kg particle has a speed of 2.00 m/s at point Ⓐ and kinetic energy of 7.50 J at point Ⓑ. What are (a) its kinetic energy at Ⓐ, (b) its speed at Ⓑ, and (c) the net work done on the particle as it moves from Ⓐ to Ⓑ?

$m = 0.6\,kg$

$V_A = 2\,m/s$

$K_B = 7.5\,J$

$V_B = ?$

$K_A = ?$

$W_{AB}\ ?$

a) $\boxed{K_A = \frac{1}{2} m V_A^2 = \frac{1}{2}\, 0.6\,kg \left(2\,\frac{m}{s}\right)^2 = 1.2\,J}$

b) $\boxed{V_B = \sqrt{\frac{2K_B}{m}} = \sqrt{\frac{2 \cdot 7.5\,J}{0.6\,kg}} = 5\,m/s}$

c) $W_{AB} = \Delta K = K_B - K_A = (7.5 - 1.2)\,J = 6.3\,J$

A 0.300-kg ball has a speed of 15.0 m/s. (a) What is its kinetic energy? (b) **What If?** If its speed were doubled, what would be its kinetic energy?

$m = 0.3\,kg$

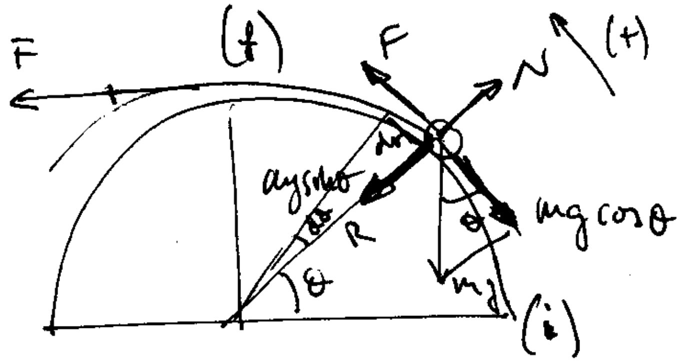
$V = 15\,m/s$

a) $K\ ? \quad = \frac{1}{2} m V^2 = \frac{1}{2}\, 0.3\,kg \cdot \left(15\,\frac{m}{s}\right)^2 = 33.8\,J$

b) $V' = 2V \rightarrow K'\ ?$

$K' = \frac{1}{2} m V'^2 = \frac{1}{2} m (2V)^2 = 4\left(\frac{1}{2} m V^2\right) = 4K$

A small particle of mass m is pulled to the top of a frictionless half-cylinder (of radius R) by a cord that passes over the top of the cylinder as illustrated in Figure P7.25. (a) Assuming the particle moves at a constant speed, show that $F = mg\cos\theta$. *Note:* If the particle moves at constant speed, the component of its acceleration tangent to the cylinder must be zero at all times. (b) By directly integrating $W = \int \vec{\mathbf{F}} \cdot d\vec{\mathbf{r}}$, find the work done in moving the particle at constant speed from the bottom to the top of the half-cylinder.



a) $v = cte$, $a = 0$ $\qquad F - mg\cos\theta = m\cancel{a}_{\,0} = 0$

$$\boxed{F = mg\cos\theta}$$

b) $$W = \int_{r_i}^{r_f} \vec{F}\, d\vec{r} = \int_{r_i}^{r_f} mg\cos\theta\, dr = \int_{\theta_i}^{\theta_f} mg\cos\theta\, R\, d\theta = mgR\int_{\theta_i}^{\theta_f} \cos\theta\, d\theta$$

$dr = R\sin(d\theta) = R\, d\theta$

$\sin(d\theta) \sim d\theta$

$\left[\sin\theta \sim \theta \quad \theta \to 0\right]$

$$W = mgR\int_0^{90} \cos\theta\, d\theta$$

$$W = mgR\left[\sin 90^\circ - \cancel{\sin 0^\circ}_{\,0}\right]$$

$$\boxed{W = mgR}$$

$\|$

store as potential energy $\boxed{W_g = mgy}$

● You can think of the work–kinetic energy theorem as a second theory of motion, parallel to Newton's laws in describing how outside influences affect the motion of an object. In this problem, solve parts (a) and (b) separately from parts (c) and (d) so that you can compare the predictions of the two theories. In a rifle barrel, a 15.0-g bullet is accelerated from rest to a speed of 780 m/s. (a) Find the work that is done on the bullet. (b) Assuming the rifle barrel is 72.0 cm long, find the magnitude of the average net force that acted on it, as $\Sigma F = W/(\Delta r \cos\theta)$. (c) Find the constant acceleration of a bullet that starts from rest and gains a speed of 780 m/s over a distance of 72.0 cm. (d) Assuming now the bullet has mass 15.0 g, find the net force that acted on it as $\Sigma F = ma$. (e) What conclusion can you draw from comparing your results?

$m = 15g \quad \Delta r = 75cm$

$V_0 = 0$

$V_f = 780 \, m/s$

a) W ?

b) $\Delta r = 72cm$ using W/K

c) a

d) F using Newton

a) $W = \int \vec{F} \, d\vec{r} = F_{av} \Delta r \cancel{\cos\theta} = F_{av} \Delta r = \Delta K = \frac{1}{2} m V_f^2 - \frac{1}{2} \cancel{m V_0^2}$

$W = \frac{1}{2} m V_f^2 = \frac{1}{2} 15 \times 10^{-3} kg \left(780 \frac{m}{s}\right)^2 = 4.56 \, kJ$

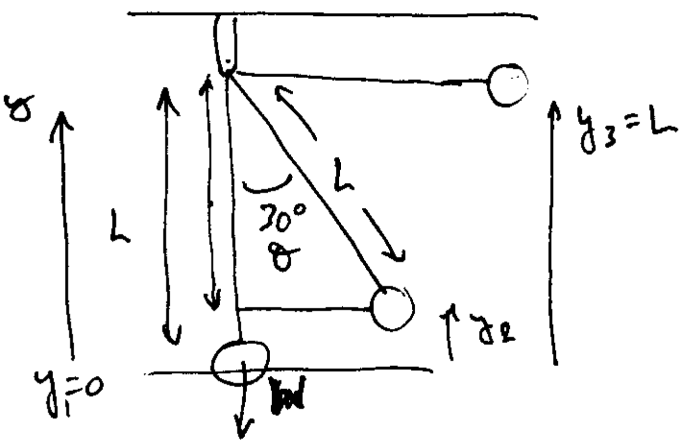
b) $F_{av} \Delta r = W \longrightarrow \boxed{F_{av} = \frac{W}{\Delta r}} = \frac{456 \, kJ}{0.72 \, m} = \boxed{6.34 \, kN}$

c) $a = \frac{V_f^2 - V_i^2}{2 \Delta r} = \frac{\left(780 \frac{m}{s}\right)^2}{2 \cdot 0.72 \, m} = 422{,}000 \frac{m}{s^2}$

the same

d) $\boxed{F_{av} = ma} = 15 \times 10^{-3} kg \cdot 422{,}000 \frac{m}{s^2} = \boxed{6.34 \, kN}$

A 400-N child is in a swing that is attached to ropes 2.00 m long. Find the gravitational potential energy of the child–Earth system relative to the child's lowest position when (a) the ropes are horizontal, (b) the ropes make a 30.0° angle with the vertical, and (c) the child is at the bottom of the circular arc.



$L = 2m$

$mg = 400\ N$

Find U when;

a) $\theta = 90°$

b) $\theta = 30°$

c) $\theta = 0$

$$U = mgy$$

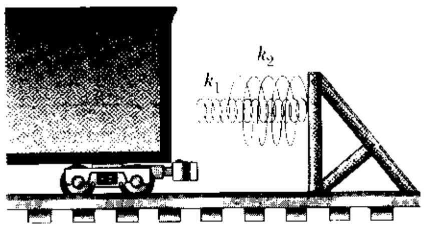
a) $U(\theta = 90°) = mgL = 800\ J$

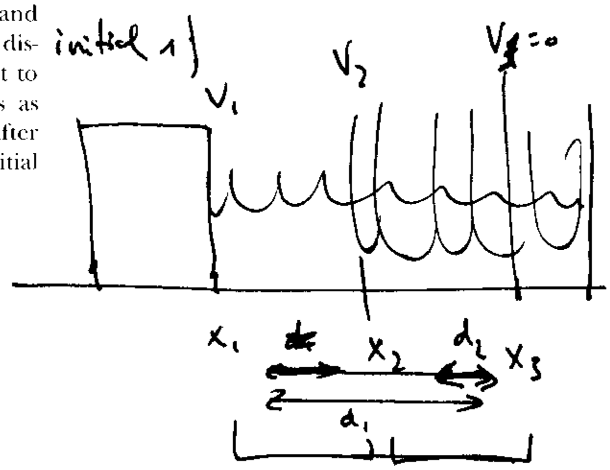
b) $U(\theta = 30°) = mg y_2 = mgL(1 - \cos\theta) = 107\ J$

$$y_2 = L - L\cos\theta = L(1 - \cos\theta)$$

c) $U(\theta = 0°) = mgL(1 - \underbrace{\cos\theta}_{1}) = 0$

A 6 000-kg freight car rolls along rails with negligible friction. The car is brought to rest by a combination of two coiled springs as illustrated in Figure P7.21. Both springs are described by Hooke's law with $k_1 = 1\ 600$ N/m and $k_2 = 3\ 400$ N/m. After the first spring compresses a distance of 30.0 cm, the second spring acts with the first to increase the force as additional compression occurs as shown in the graph. The car comes to rest 50.0 cm after first contacting the two-spring system. Find the car's initial speed.





$x_1 = 0$

$x_2 = 30\,cm = 0.3\,m$

$x_3 = 0.5\,m$

$m = 6\,kg$

$k_1 = 1600\ N/m$

$k_2 = 3400\ N/m$

~~ΔK_1~~ ~~ΔK_2~~

" "

W_1 W_2

d_1 d_2

$\underbrace{\qquad\qquad}_{W_T}$

ΔK

$$W_{TOTAL} = W_1 + W_2 = \Delta K$$

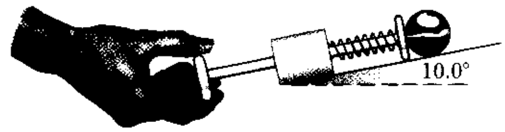
$$\cancel{\tfrac{1}{2}} k_1 d_1^2 + \cancel{\tfrac{1}{2}} k_2 d_2^2 = \tfrac{1}{2} m \cancelto{0}{v_f^2} - \cancel{\tfrac{1}{2}} m v_i^2$$

$$V_1 = \sqrt{\frac{k_1 d_1^2 + k_2 d_2^2}{m}} = 0.299\ \frac{m}{s}$$

$d_1 = x_3 - x_1 = 0.5\,m$

$d_2 = x_3 - x_2 = 0.2\,m$

The ball launcher in a pinball machine has a spring that has a force constant of 1.20 N/cm (Fig. P7.57). The surface on which the ball moves is inclined 10.0° with respect to the horizontal. The spring is initially compressed 5.00 cm. Find the launching speed of a 100-g ball when the plunger is released. Friction and the mass of the plunger are negligible.



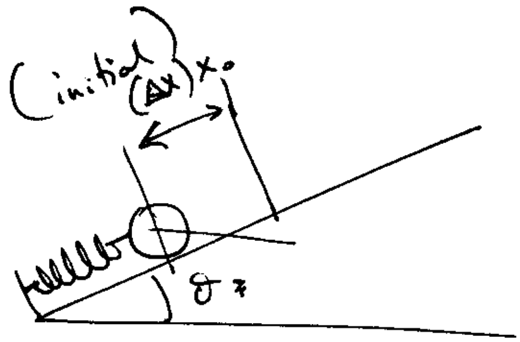
$k = 1.20\ N/cm = 120\ \frac{N}{m}$

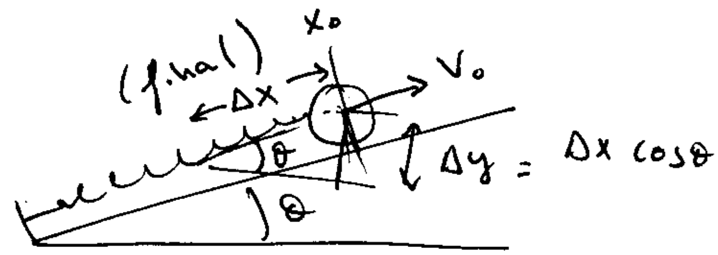
$\theta = 10°$

$\Delta x = 5cm = 0{,}05\ m$

$m = 100g = 0{,}1\ kg$

V ?





initial) $K_i = 0$

final) $K_f = \frac{1}{2} m V^2$ $\Big\}$ $\Delta K = \frac{1}{2} m V^2$

$$W_T = W_{sp} + W_g = \Delta K$$

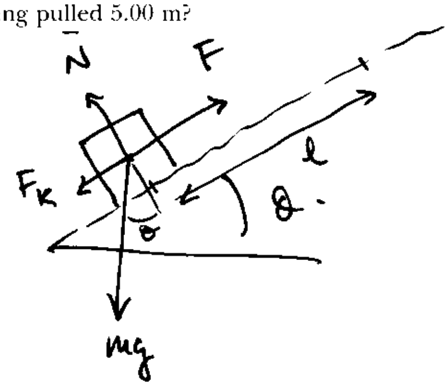
$$W_{sp} = \frac{1}{2} k(\Delta x)^2$$

$$W_g = mg\,\Delta y = mg\,\Delta x \cos\theta$$

$$\frac{1}{2} k \Delta x^2 + mg\,\Delta x \cos\theta = \frac{1}{2} m V^2$$

$$V = \sqrt{\left(\frac{k}{m}\Delta x + 2g\cos\theta\right)\Delta x} = 1.68\ m/s$$

A crate of mass 10.0 kg is pulled up a rough incline with an initial speed of 1.50 m/s. The pulling force is 100 N parallel to the incline, which makes an angle of 20.0° with the horizontal. The coefficient of kinetic friction is 0.400, and the crate is pulled 5.00 m. (a) How much work is done by the gravitational force on the crate? (b) Determine the increase in internal energy of the crate–incline system owing to friction. (c) How much work is done by the 100-N force on the crate? (d) What is the change in kinetic energy of the crate? (e) What is the speed of the crate after being pulled 5.00 m?



$m = 10\,kg$

$V_i = 1.5\,m/s$

$F = 100\,N$

$\theta = 20°$

$\mu_k = 0.4$

$\ell = 5\,m$

a) W_g ?

b) ΔE_{int} ?

c) W_F ?

d) ΔK ?

e) V_f ?

a) $\boxed{W_g = \vec{W} \cdot \vec{\ell} = mg\,\ell \cos(110°) = -168\,J}$

b) $\Delta E = W_{NC} = \vec{F}_k \cdot \vec{\ell} = \mu_k \vec{N} \cdot \vec{\ell} = \underbrace{\mu_k\, mg\cos\theta}\, \underbrace{\ell}\, \underbrace{\cos(180°)}$

$\vec{N} = mg\cos\theta$

$\boxed{\Delta E_{int} = -\Delta E = 184\,J}$

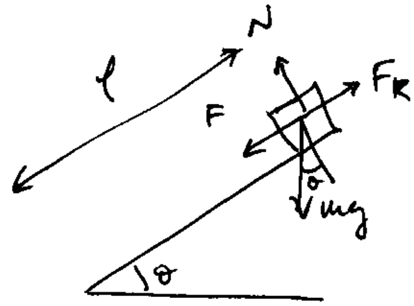
c) $\boxed{W_F = \vec{F} \cdot \vec{\ell} = F\ell\cos 0° = F\ell = 500\,J}$

d) $\Delta K = W_T = W_g + W_{NC} = W_g + W_F + W_K = -168\,J + 500\,J - 184\,J$

$\boxed{\Delta K = 148\,J}$

e) $\Delta K = 148\,J = \frac{1}{2} m V_f^2 - \frac{1}{2} m V_i^2 \rightarrow V_f = \sqrt{\frac{2\Delta K}{m} + V_i^2} = 5.65\,\frac{m}{s}$

A boy in a wheelchair (total mass 47.0 kg) wins a race with a skateboarder. The boy has speed 1.40 m/s at the crest of a slope 2.60 m high and 12.4 m long. At the bottom of the slope his speed is 6.20 m/s. Assume air resistance and rolling resistance can be modeled as a constant friction force of 41.0 N. Find the work he did in pushing forward on his wheels during the downhill ride.



$m = 47\,kg$

$y_i = 2.6\,m$

$y_f = 0$

$\ell = 12.4\,m$

$v_f = 6.2\,m/s$

$v_i = 1.40\,m/s$

$F_K = 41\,N$

W_{CH}

$\Delta K = W_T$

$\boxed{\Delta E = W_{NC}}$

$W_T = W_C + W_{NC}$

$W_C = W_g = mg\ell\sin\theta$

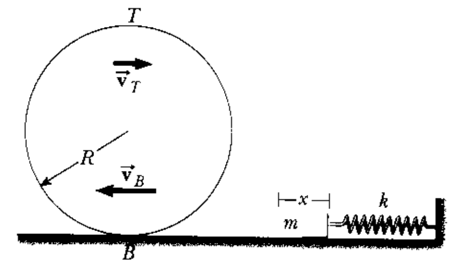
$W_{NC} = W_K + W_F$

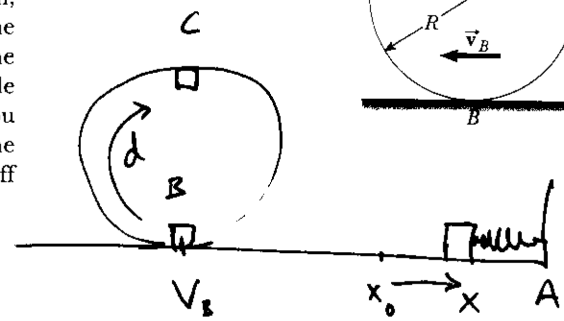
$\Delta E = W_{NC} = W_K + W_F$

$$\underbrace{\frac{1}{2}m\left[v_f^2 - v_i^2\right]}_{\Delta K} \underbrace{- mgy_i}_{+\ \Delta U} = \underbrace{F_K \ell \underbrace{\cos(180^\circ)}_{-1}}_{W_K} + W_F$$

$$\boxed{W_F = \frac{1}{2}m\left[v_f^2 - v_i^2\right] - mgy_i + F_K\ell = 168\,J}$$

● ▲ A block of mass 0.500 kg is pushed against a horizontal spring of negligible mass until the spring is compressed a distance x (Fig. P8.59). The force constant of the spring is 450 N/m. When it is released, the block travels along a frictionless, horizontal surface to point B, the bottom of a vertical circular track of radius R = 1.00 m, and continues to move along the track. The speed of the block at the bottom of the track is v_B = 12.0 m/s, and the block experiences an average friction force of 7.00 N while sliding up the track. (a) What is x? (b) What speed do you predict for the block at the top of the track? (c) Does the block actually reach the top of the track, or does it fall off before reaching the top?





$\cancel{M} = 0.5 Kg$

$k = 450 N/m$

$R = 1 m$

$V_B = 12 m/s$

$F_K = 7N$

a) x ?

b) V_{Top} ?

a) $\Delta E = 0$, $U_A = K_B$

$$\frac{1}{2} k x^2 = \frac{1}{2} m v^2$$

$$\boxed{x = \sqrt{\frac{m}{k}}\, v = 0.4 m}$$

b) $\Delta E = W_{NC} = \vec{F}_k \cdot \Delta \vec{r} = -F \cdot d$

$E_c - E_B = -Fd \qquad d = \pi R$

$$mg\underbrace{y_c}_{2R} + \frac{1}{2} m v_c^2 - mg\cancel{y_B}^{\,0} - \frac{1}{2} m v_B^2 = -\pi R F_K$$

$$\boxed{V_c = 4.10 m/s}$$

c) It falls if $a_c < g$

$a_c = \frac{V_c^2}{R} = 16.8 m/s^2 > g$, ~~it~~ stays on the track.