LINEAR MOMENTUM AND COLLISIONS

CONSERVATION OF LINEAR MOMENTUM

Let's assume the collision of two objects (m_1, m_2), under the view of Newton's third law;

$$\vec{F}_{12} = -\vec{F}_{21} \longrightarrow \vec{F}_{12} + \vec{F}_{21} = 0$$

We know from Newton's second law that;

$$m_1 \vec{a}_1 + m_2 \vec{a}_2 = 0$$

which is;

$$m_1 \frac{d\vec{v}_1}{dt} + m_2 \frac{d\vec{v}_2}{dt} = 0$$

or:

$$\frac{d}{dt}\left(m_1 \vec{v}_1 + m_2 \vec{v}_2 \right) = 0$$

Therefore: $m_1 \vec{v}_1 + m_2 \vec{v}_2 = ct$

As we will see, in the absence of external forces, the linear momentum of a system is conserved.

$$\boxed{\vec{p} = m\vec{v}}$$ linear momentum.

Total (net) linear momentum:

$$\vec{P}_T \equiv \vec{P}_N = \sum_i \vec{P}_i = \sum_i m_i \vec{v}_i$$

$$\left.\begin{aligned} P_{Tx} &= \sum_i P_{ix} \\ P_{Ty} &= \sum_i P_{iy} \end{aligned}\right\} \quad \text{where} \quad \begin{aligned} P_x &= m v_x \\ P_y &= m v_y \end{aligned}$$

Newton's second law:

$$\sum \vec{F} = m\vec{a} = m\frac{d\vec{v}}{dt} = \frac{d}{dt}(m\vec{v}) = \frac{d\vec{p}}{dt}$$

$$\boxed{\sum \vec{F} = \frac{d\vec{p}}{dt}}$$

Newton formulated it like that.

This is more general, since it allows to study the case in which mass changes with time.

Conservation of linear momentum:

$$\sum \vec{F} = \underbrace{\sum \vec{F}_{int}}_{\substack{= \\ 0 \\ \text{Newton's 3rd}}} + \sum F_{ext} = \frac{d\vec{P}_T}{dt}$$

Therefore; if $\sum F_{ext} = 0$ then;

$$\boxed{\vec{P}_T = \text{constant}}$$

IMPULSE AND MOMENTUM

As seen before, when a net force changes with time, there will be a change in linear momentum $\Delta\vec{p}$.
This change is called "Impulse" (I);

$$\boxed{\vec{I} = \Delta\vec{p} = \vec{p}_f - \vec{p}_i = \int_{t_i}^{t_f} \vec{F}_N \, dt = \int_{t_i}^{t_f} \sum_i \vec{F}_i \, dt}$$

When working with average force $\left(\sum \vec{F}_{avg}\right)$, then

$$\boxed{\vec{I} = \Delta\vec{p} = \left(\sum F\right)_{avg} \Delta t}$$ where $\Delta t = t_f - t_i$

COLLISIONS IN ONE DIMENSION

Elastic collisions : $\Delta K = 0$, $\Delta\vec{p} = 0$

Inellastic collisions : $\Delta K \neq 0$ $\Delta\vec{p} = 0$

Perfectly inelastic collisions (when the objects stick together)

$$V_{1f} = V_{2f} \equiv V$$

Let's see 1-D collisions: $\left(\vec{v} \rightarrow v\right)$

* PERFECTLY INELASTIC COLLISIONS (SIMPLEST CASE)

$$\Delta \vec{p},\ \Delta K \neq 0 \qquad v_{1f} = v_{2f} = v_f$$

$$\downarrow$$

$$p_i = p_f \longrightarrow \boxed{m_1 v_{1i} + m_2 v_{2i} = (m_1 + m_2)\, v_f}$$

$$\boxed{v_f = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2}}$$

** ELASTIC COLLISIONS:

$$\Delta p = 0 \quad , \quad \Delta K = 0$$

$$\boxed{m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}}$$

$$\downarrow$$

$$\boxed{\frac{1}{2} m_1 v_{1i}^2 + \frac{1}{2} m_2 v_{2i}^2 = \frac{1}{2} m_2 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2}$$

complicated situation.

$$\boxed{\begin{aligned} v_{1f} &= \left(\frac{m_1 - m_2}{m_1 + m_2}\right) v_{1i} + \left(\frac{2 m_2}{m_1 + m_2}\right) v_{2i} \\ v_{2f} &= \left(\frac{2 m_1}{m_1 + m_2}\right) v_{1i} + \left(\frac{m_2 - m_1}{m_1 + m_2}\right) v_{2i} \end{aligned}}$$

Some special situations:

(A) $m_1 = m_2 \longrightarrow V_{1f} = V_{2i}$ and $V_{2f} = V_{1i}$ (billiard)

(B) $V_{2i} = 0 \longrightarrow$

$$V_{1f} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) V_{1i}$$

$$V_{2f} = \left(\frac{2m_1}{m_1 + m_2}\right) V_{1i}$$

$\hookrightarrow$ and $m_1 = m_2$: $V_{1f} = 0$, $V_{2f} = V_{1i}$ (hanging balls)

(C) $m_1 \gg m_2$ and $V_{2i} = 0 \longrightarrow$

$$V_{1f} \approx V_{1i}$$

$$V_{2f} \approx V_{1i}$$

$m_1 \ll m_2$ and $V_{2i} = 0 \longrightarrow$

$$V_{1f} \approx -V_{1i}$$

$$V_{2f} \approx 0$$

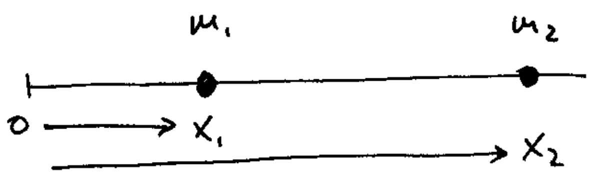
COLLISIONS IN TWO DIMENSIONS

<u>Ellastic</u>: $\Delta\vec{p}=0, \Delta K=0 \quad \Delta p_x = 0, \Delta p_y = 0$

<u>Innellastic</u>: $\Delta\vec{p} = 0, \Delta K \neq 0$

THE CENTER OF MASS

a) Let's consider two particles:



the center of mass is at $\boxed{X_{CM} = \dfrac{m_1 x_1 + m_2 x_2}{m_1 + m_2}}$

It is like having a mass $(M = m_1 + m_2)$ at a position (X_{CM})

b) For a system of many particles:

$$\boxed{X_{CM} = \frac{1}{M}\sum_i m_i x_i} \quad \text{where} \quad \boxed{M = \sum m_i}$$

c) In more than one dimension:

$$y_{CM} = \frac{1}{M}\sum_i m_i y_i \quad \text{and} \quad z_{CM} = \frac{1}{M}\sum m_i z_i$$

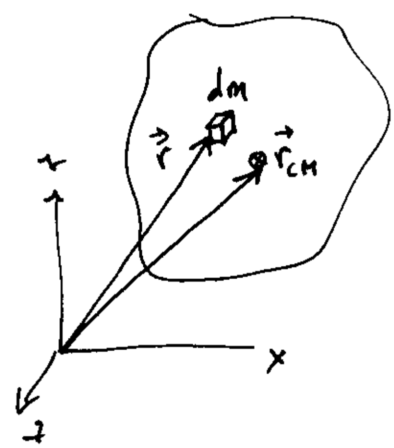
d) in general:

$$\vec{r}_{CM} = x_{cm}\,\hat{i} + y_{cm}\,\hat{j} + z_{cm}\,\hat{z}$$

$$\boxed{\vec{r}_{CM} = \frac{1}{M}\sum_i m_i \vec{r}_i}$$

e) Center of mass of a continuous object:

$$\vec{r}_{CM} = \frac{1}{M}\int \vec{r}\, dm$$



9.6. MOTION OF A SYSTEM OF PARTICLES

We could analyze the motion of a system of particles by looking at its center of mass.

$$\vec{x}_{CM} = \frac{1}{M}\sum_i m_i \vec{r}_i$$

Velocity: $\vec{V}_{cm} = \dfrac{d\vec{r}_{cm}}{dt} = \dfrac{1}{M}\sum m_i \vec{v}_i$

Linear momentum: $\vec{P} = M\vec{V}_{cm} = \sum_i m_i \vec{v}_i$

Acceleration: $\vec{a}_{CM} = \frac{d\vec{v}_{CM}}{dt} = \frac{1}{M} \sum_i m_i \vec{a}_i$

Newton's second law: $M\vec{a}_{CM} = \sum_i m_i \vec{a}_i = \sum_i \vec{F}_i$

(where $\vec{F}_i$ is the net force on particle i)

As we have seen before:

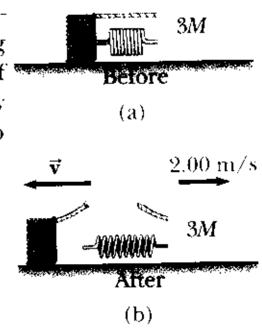
$$\sum \vec{F} = \underbrace{\cancel{\sum F_{int}}}_{0} + \sum F_{ext}$$

therefore, the acceleration of the center of mass of a system is only determined by external forces.

$$\boxed{M\vec{a}_{cm} = \sum \vec{F}_{ext}}$$

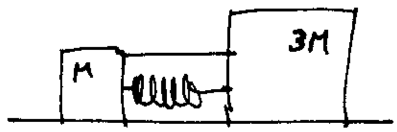
[We won't see sections 9.7 and 9.8]

● Two blocks of masses M and $3M$ are placed on a horizontal, frictionless surface. A light spring is attached to one of them, and the blocks are pushed together with the spring between them (Fig. P9.4). A cord initially holding the blocks together is burned; after that happens, the block of mass $3M$ moves to the right with a speed of 2.00 m/s. (a) What is the velocity of the block of mass M? (b) Find the system's original elastic potential energy, taking $M = 0.350$ kg. (c) Is the original energy in the spring or in the cord? Explain your answer. (d) Is momentum of the system conserved in the bursting-apart process? How can it be, with large forces acting? How can it be, with no motion beforehand and plenty of motion afterward?

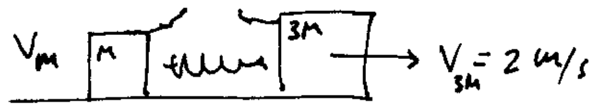


(a) $\Delta p = 0$

$p_i = p_f$



$p_i = 0$ $\longrightarrow x$



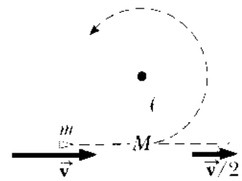
$$p_f = M V_M + 3M V_{3M}$$

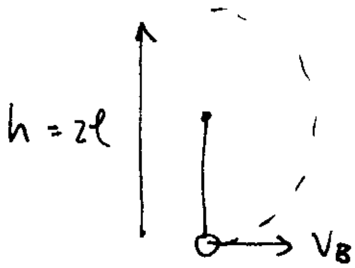
$$\boxed{V_M = -\frac{3M V_{3M}}{M} = -3V_{3M} = -6 \text{ m/s}}$$

(b) Energy $E_i = \frac{1}{2}kx^2 = E_f = \frac{1}{2}M V_M^2 + \frac{3}{2}M V_{3M}^2 = 8.40$ J

Energy ~~f~~ was stored by the spring.

As shown in Figure P9.18 (page 262), a bullet of mass m and speed v passes completely through a pendulum bob of mass M. The bullet emerges with a speed of $v/2$. The pendulum bob is suspended by a stiff rod of length ℓ and negligible mass. What is the minimum value of v such that the pendulum bob will barely swing through a complete vertical circle?





$\frac{1}{2} \cancel{m} V_B^{*2} = \cancel{m} g 2\ell$

$\boxed{V_B^* = \sqrt{2g2\ell} = \sqrt{4g\ell} = 2\sqrt{g\ell}}$

(before)

m_b m_B

V_{b_i} $V_{B_i} = 0$

(after)

m_B V_{Bf}

m_b

V_{bf}

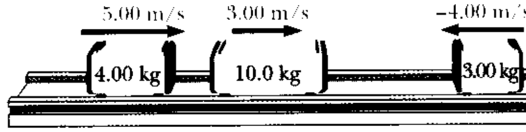
$\Delta p = 0 \qquad p_f - p_i = 0$

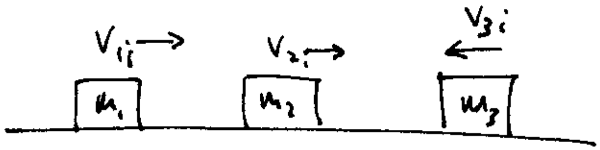
$m_b V_{bf} + m_B V_{Bf} - m_b V_{b_i} = 0$

$V_{Bf} \equiv V_B^* = 2\sqrt{g\ell} \qquad V_{bf} = \frac{V_{b_i}}{2}$

$$\boxed{V_{b_i} = \frac{4 M_B}{m_b}\sqrt{g\ell}}$$

● (a) Three carts of masses 4.00 kg, 10.0 kg, and 3.00 kg move on a frictionless, horizontal track with speeds of 5.00 m/s, 3.00 m/s, and 4.00 m/s as shown in Figure P9.24. Velcro couplers make the carts stick together after colliding. Find the final velocity of the train of three carts. (b) **What If?** Does your answer require that all the carts collide and stick together at the same moment? What if they collide in a different order?





$m_1 = 4k$

$V_{1i} = 5\,m/s$

$m_2 = 10\,kg$

$V_{2i} = 3\,m/s$

$m_3 = 3\,kg$

$V_{3i} = -4\,m/s$

a) $\Delta P = 0 \qquad P_f = P_i$

$$P_f = (m_1 + m_2 + m_3)V_f = P_i = m_1 V_1 + m_2 V_2 + m_3 V_3$$

$$\boxed{V_f = \frac{m_1 V_1 + m_2 V_2 + m_3 V_3}{m_1 + m_2 + m_3} = +2.24\,m/s}$$

(b) No, they do not need to collide at the same time.

Let's assume m_2 and m_3 collide first.

$$P_f = (m_2 + m_3)V_{23} = m_2 V_2 + m_3 V_3 = P_i$$

$$V_{23} = +1.38\,m/s$$

Then $(m_2 + m_3)$ collide with m_1

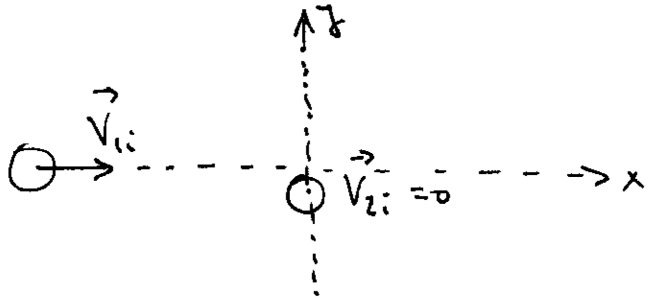
$$P_f = (m_1 + m_2 + m_3)V_{123} = (m_2 + m_3)V_{23} + m_1 V_1 = P_i$$

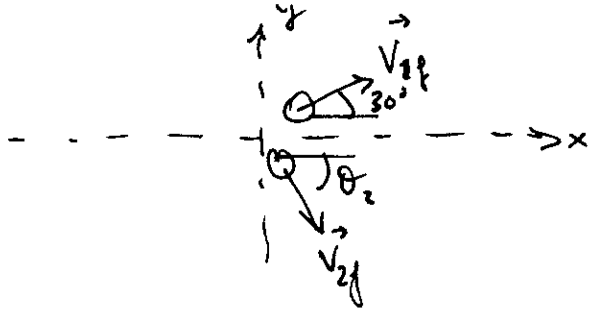
$$\boxed{V_{123} = +2.24\,\frac{m}{s} = V_f}$$

A billiard ball moving at 5.00 m/s strikes a stationary ball of the same mass. After the collision, the first ball moves, at 4.33 m/s, at an angle of 30.0° with respect to the original line of motion. Assuming an elastic collision (and ignoring friction and rotational motion), find the struck ball's velocity after the collision.

$\vec{V}_{1i} = V_{1x} = 5\,m/s$ $\qquad V_{1f} = 4.33\,m/s$ at $\theta_1 = 30°$

$\vec{V}_{2i} = 0$ $\qquad \boxed{\vec{V}_{2f}}$?





$\Delta\vec{p} = 0$

$\Delta p_x = 0 \quad \Delta p_y = 0$

$\downarrow$

x) $P_{fx} - P_{ix} = m_1 V_{1f}\cos 30° - m_2 V_{2fx} = 0$ $\quad \boxed{V_{2fx} = 1.25\,m/s}$

y) $P_{fy} - P_{iy} = m_1 V_{1f}\sin 30° - (-m_2 V_{2fy}) = 0$ $\quad \boxed{V_{2fy} = -2.16\,m/s}$

$$\boxed{V_{2f} = \sqrt{V_{2fx}^2 + V_{2fy}^2} = 2.50\,m/s}$$

$$\boxed{\theta_2 = \tan^{-1}\left(\frac{V_{2fy}}{V_{2fx}}\right) = -60°}$$

▲ A 12.0-g wad of sticky clay is hurled horizontally at a 100-g wooden block initially at rest on a horizontal surface. The clay sticks to the block. After impact, the block slides 7.50 m before coming to rest. If the coefficient of friction between the block and the surface is 0.650, what was the speed of the clay immediately before impact?

$m = 12\text{ g}$

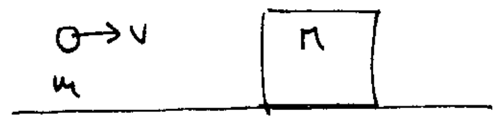
$M = 100\text{ g}$

$d = 7.5\text{ m}$

$\mu_k = 0.65$

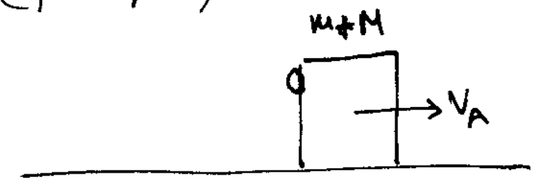
v ?

(before)

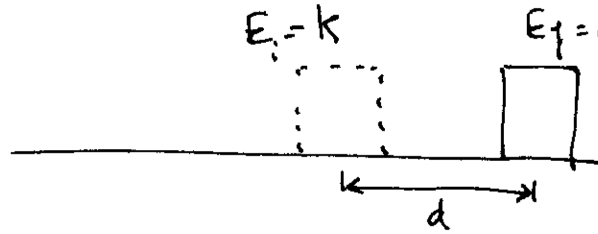


$P_{before} = mv$

(just after)



$P_{after} = (m+M)V_A$

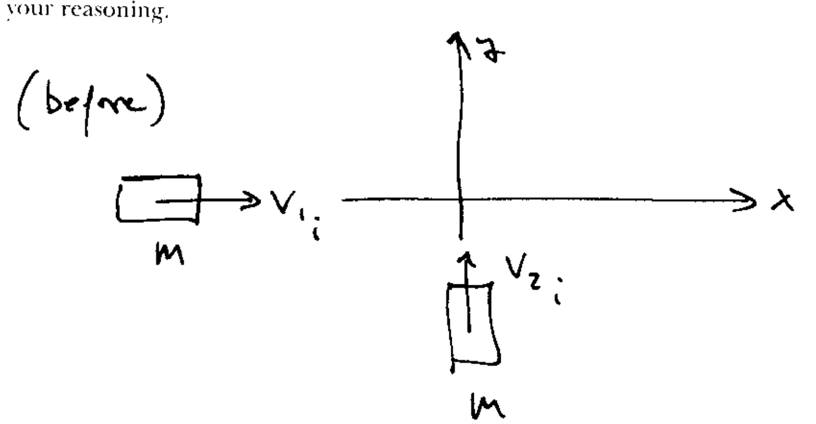


$$\Delta E = E_f - E_i = -\frac{1}{2}(M+m)V_A^2 = W_k = -(M+m)\,g\,d$$

$$\boxed{V_A = 9.77\ \text{m/s}}$$

$$\boxed{V = \frac{m+M}{m}V_A = 91.2\ \frac{\text{m}}{\text{s}}}$$

● Two automobiles of equal mass approach an intersection. One vehicle is traveling with velocity 13.0 m/s toward the east, and the other is traveling north with speed v_{2i}. Neither driver sees the other. The vehicles collide in the intersection and stick together, leaving parallel skid marks at an angle of 55.0° north of east. The speed limit for both roads is 35 mi/h, and the driver of the northward-moving vehicle claims he was within the speed limit when the collision occurred. Is he telling the truth? Explain your reasoning.

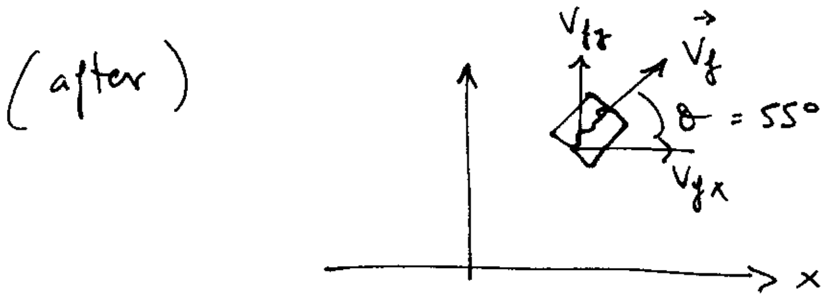


$\theta = 55°$

$V_1 = 13$ mi/h

~~$V_2 = 35$ m/s~~

$V_2 < 35$ mi/h ?



$$\Delta P_x = 0 \rightarrow P_{xi} = P_{xf} \rightarrow \cancel{m}V_1 = 2mV_{fx} = 2\cancel{m}V_f\cos\theta$$
$$\Delta P_y = 0 \rightarrow P_{yi} = P_{yf} \rightarrow \cancel{m}V_2 = 2mV_{fy} = 2\cancel{m}V_f\sin\theta$$

$$V_1 = 2V_f\cos\theta$$
$$V_2 = 2V_f\sin\theta$$

$$\frac{V_1}{V_2} = \frac{\cos\theta}{\sin\theta}$$

$$V_2 = V_1\tan\theta = 13\,\frac{m}{s}\tan 55° = 18.6\,\frac{m}{s}$$

$$\left(V_2 = 41.5\text{ mi/h} \gg \text{speed limit}\right)$$