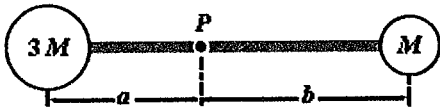


NAME SOLUTIONS

P.1 (25 points): The rigid body shown is rotated about an axis perpendicular to the paper and through the point P. If $M = 0.40$ kg, $a = 30$ cm, and $b = 50$ cm, how much work is required to take the body from rest to an angular speed of 5.0 rad/s? Neglect the mass of the connecting rods and treat the masses as particles.



Work-energy theorem:

$$\Delta K = W \quad \Delta K = K_f - K_i = K_f$$

\uparrow
($K_i = 0$ at rest)

Rotational kinetic energy:

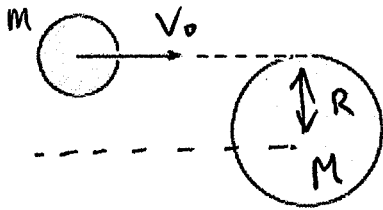
$$K_R = \frac{1}{2} I_{\text{Total}} \omega^2 = \frac{1}{2} (I_{3M} + I_M) \omega^2$$

All reduces to calculate the moments of inertia of the two particles: $I = mr^2$

$$K_F = \frac{1}{2} [3Ma^2 + Mb^2] \omega^2 = \frac{1}{2} [3 \times 0.4 \text{ kg} \cdot (0.3 \text{ m})^2 + 0.4 \text{ kg} \cdot (0.5 \text{ m})^2] (5 \frac{\text{rad}}{\text{s}})^2$$

$$W = K_F = 2.6 \text{ J}$$

P.2 (25 points): A particle of mass $m = 0.10$ kg and speed $v_0 = 5.0$ m/s collides and sticks to the end of a uniform solid cylinder of mass $M = 1.0$ kg and radius $R = 20$ cm. If the cylinder is initially at rest and is pivoted about a frictionless axle through its center, what is the final angular velocity (in rad/s) of the system after the collision?



~~Use~~

Conservation of angular momentum

$$L_i = L_f$$

Initial angular momentum:

$$L_i = \vec{r} \times \vec{p} = R m v_0$$

Final angular momentum:

$$L_f = I_{\text{TOTAL}} \omega = \left(m R^2 + \frac{1}{2} M R^2 \right) \omega$$

$$\left\{ I_{\text{TOTAL}} = I_m + I_{\text{cylinder}} = m R^2 + \frac{1}{2} M R^2 \right\}$$

Making the angular momenta equal:

$$R m v_0 = \left(m R^2 + \frac{1}{2} M R^2 \right) \omega$$

and solving for ω :

$$\omega = \frac{R m v_0}{\left(m R^2 + \frac{1}{2} M R^2 \right)} = 8.3 \text{ rad/s}$$

P.3 (25 points): How long must a pendulum be on the Moon, where $g = 1.6 \text{ N/kg}$, to have a period of 2.0 s?

this is really easy!

$$T = \frac{1}{f} \quad \left(f = \frac{\omega}{2\pi} \right) \quad T = \frac{2\pi}{\omega}$$

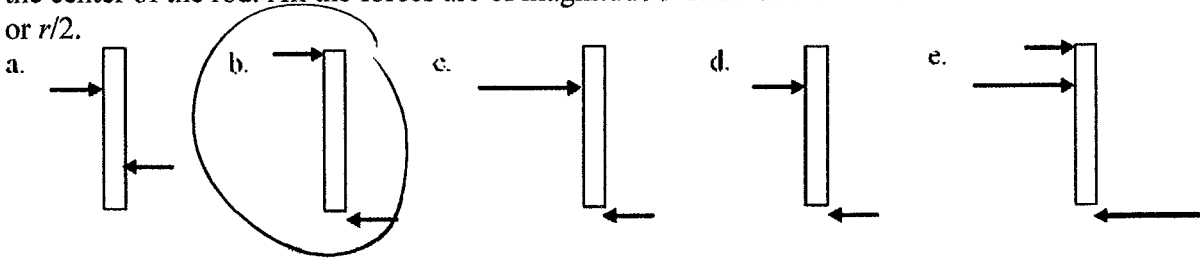
$$\omega = \sqrt{\frac{g}{L}} \rightarrow T = 2\pi \sqrt{\frac{L}{g}} \rightarrow \left(\frac{T}{2\pi} \right)^2 = \frac{L}{g} \rightarrow L = g \left(\frac{T}{2\pi} \right)^2$$

$$L = 1.6 \frac{\text{N}}{\text{kg}} \left(\frac{2\text{s}}{2\pi} \right)^2 = 0.16 \text{ m}$$

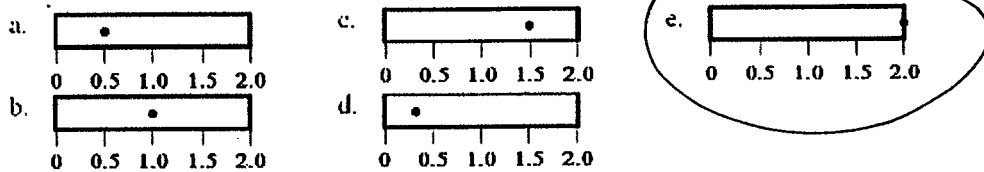
Q1 (5 points) The angular speed of the hour hand of a clock, in rad/min, is

- a. $\frac{1}{1800} \pi$.
- b. $\frac{1}{60} \pi$.
- c. $\frac{1}{30} \pi$.
- d. π .
- e. 120π .

Q2 (5 points) Which of the following diagrams shows the greatest magnitude net torque with a zero net force? All the rods, of length $2r$, rotate about an axis that is perpendicular to the rod and fixed in the center of the rod. All the forces are of magnitude F or $2F$ and all distances from the axis are r or $r/2$.



Q3 (5 points) The diagram below shows five 20-kg rods of the same 2.0-m length free to rotate about axes through the rods, as indicated. Which rod experiences the greatest magnitude gravitational torque?



Q4 (5 points) A solid sphere, spherical shell, solid cylinder and a cylindrical shell all have the same mass m and radius R . If they are all released from rest at the same elevation and roll without slipping, which reaches the bottom of an inclined plane first?

- a. solid sphere
- b. spherical shell
- c. solid cylinder
- d. cylindrical shell
- e. all take the same time

Q5 (5 points) What makes the periodic motion of a mass attached to a string to change its frequency?

- a. Increasing the time during which the system oscillates
- b. Hanging the system from the ceiling
- c. Changing the length of the spring without changing its spring constant
- d. Friction
- e. Larger gravitational acceleration