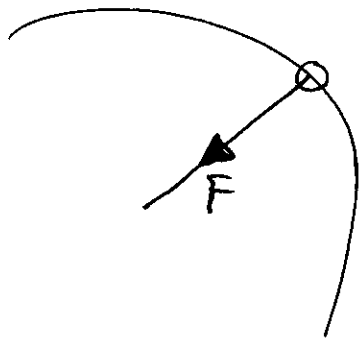
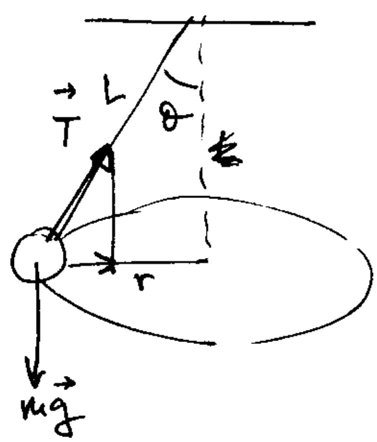
CIRCULAR MOTION AND OTHER

NEWTON'S SECOND LAW FOR UNIFORM CIRCULAR MOTION



$$\boxed{\sum F_{radial} = ma_c = m\frac{v^2}{r}}$$

we apply this to the next situation:



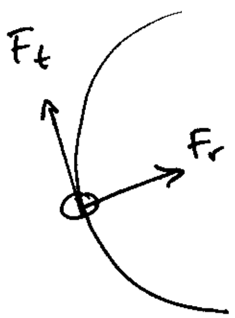
$$\sum F_{rad} = m\frac{v^2}{r}$$

$$\begin{cases} T_x = T\sin\theta = m\dfrac{v^2}{r} \\ T_y = T\cos\theta - mg = 0 \end{cases}$$

$$\tan\theta = \frac{v^2}{rg} \rightarrow \boxed{v = \sqrt{rg\tan\theta}}$$

$$r = L\sin\theta \longrightarrow \boxed{v = \sqrt{Lg\sin\theta\tan\theta}}$$

NON-UNIFORM CIRCULAR MOTION

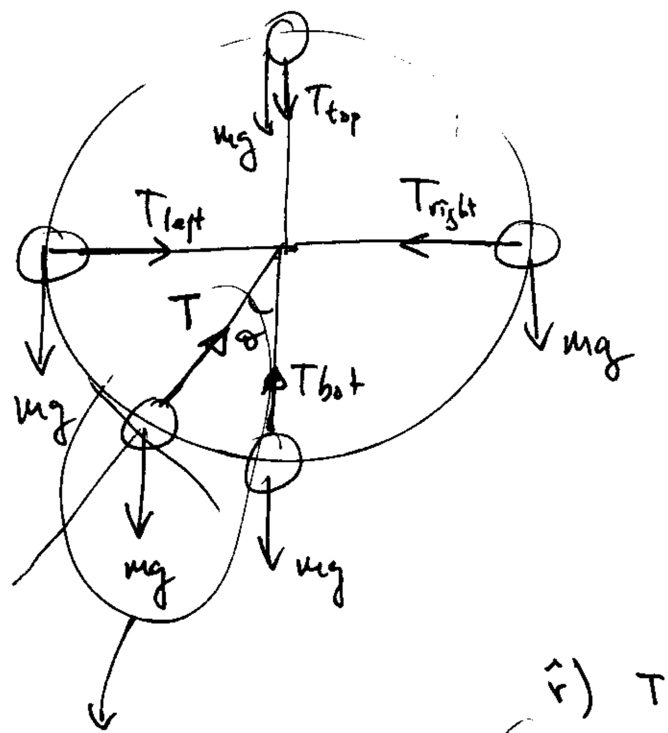


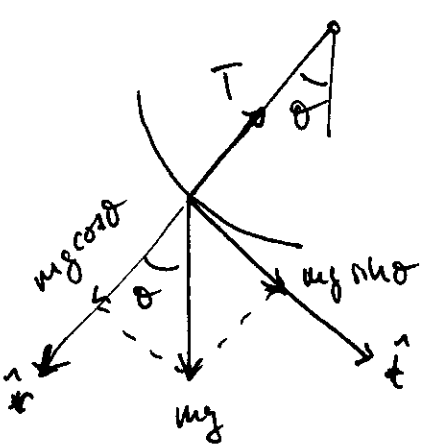
$$\vec{F}_N = F_r\,\hat{r} + F_t\,\hat{t}$$

$$\vec{a} = \vec{a}_c\,\hat{i} + \vec{a}_t\,\hat{t}$$

$$a = \sqrt{a_c^2 + a_t^2}$$

clearer example:



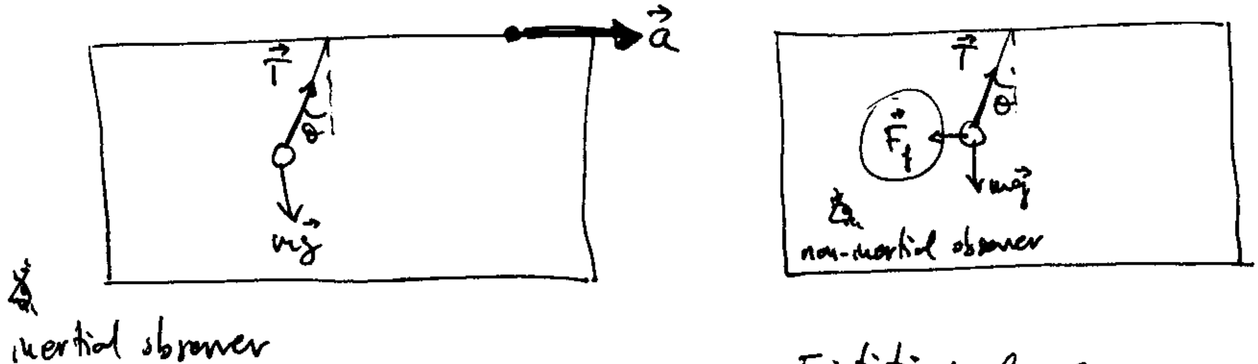


$\hat{r}$) $T - mg\cos\theta = m\dfrac{v^2}{r} = m a_c$

$\hat{t}$) $mg\sin\theta = m a_t \rightarrow a_t = g\sin\theta$

$$\boxed{T = mg\left(\frac{v^2}{rg} + \cos\theta\right)}$$

MOTION IN ACCELERATED FRAMES



Fictitious force.

x) $T\sin\theta = ma$

y) $T\cos\theta - mg = 0$

x') $T\sin\theta - F_f = 0$

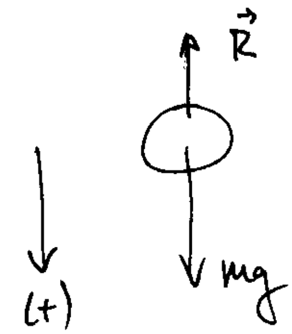
y') $T\cos\theta - mg = 0$

$$\boxed{F_f = ma}$$

MOTION WITH RESISTIVE FORCES

① $\vec{R} = -b\vec{v}$

An object falling inside a fluid.



$$mg - \vec{R} = ma$$

$$mg - bv = ma = m\frac{dv}{dt}$$

we rewrite the expression as;

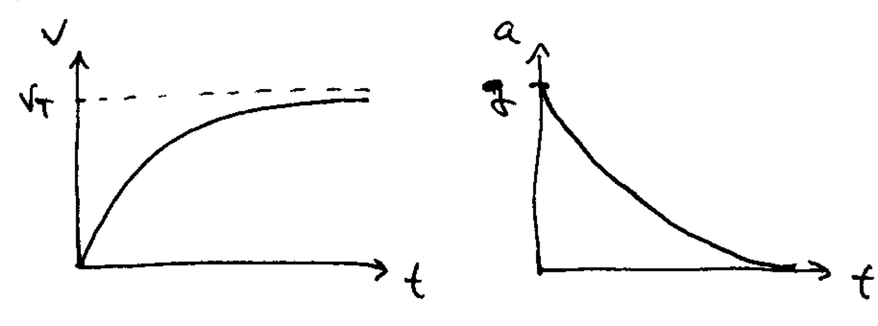
$$\frac{dv}{dt} = g - \frac{b}{m}v \qquad \text{First order differential equation}$$

its solution is:

$$\boxed{v = \frac{mg}{b}\left(1 - e^{-bt/m}\right) \equiv V_T\left(1 - e^{-t/\tau}\right)}$$

and the acceleration:

$$\boxed{a = \frac{dv}{dt} = \frac{d}{dt}\left[\frac{mg}{b}\left(1 - e^{-bt/m}\right)\right] = g\,e^{-bt/m}}$$



② $\vec{R} \propto A v^2$ high speed objects moving in fluids.

$$R = \frac{1}{2} D \rho A v^2$$

D ≡ drag coefficient (0.5 → 2) sphere irregular

ρ ≡ density of fluid (air)

A ≡ cross-sectional area of the object.

$$\sum F = mg - \frac{1}{2} D\rho A v^2 = ma$$

$$a = g - \left(\frac{D\rho A}{2m}\right) v^2 \qquad \frac{dv}{dt} = g - \left(\frac{D\rho A}{2m}\right) v^2$$

second order differential equation

We can find the terminal speed by making $a = 0$

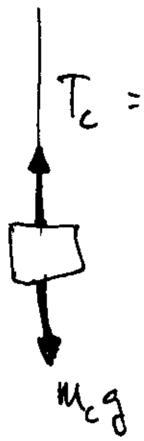
$$\boxed{V_T = \sqrt{\frac{2mg}{D\rho A}}}$$

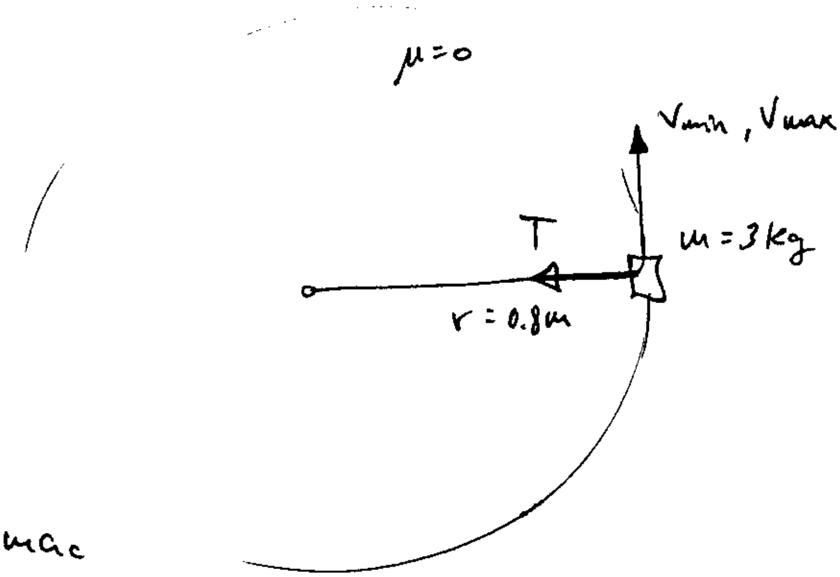
TERMINAL SPEED

skydiver	$m = 75$ kg	$A = 0.70\ m^2$	$V_T = 60$ m/s
baseball	0.145	4.2×10^{-3}	$V_T = 43$ m/s
golfball	0.046	1.4×10^{-3}	$V_T = 44$
raindrop	3×10^{-5}	1.3×10^{-5}	$V_T = 9$ m/s

A light string can support a stationary hanging load of 25.0 kg before breaking. A 3.00-kg object attached to the string rotates on a horizontal, frictionless table in a circle of radius 0.800 m, and the other end of the string is held fixed. What range of speeds can the object have before the string breaks?

$$T_c = m_c g = 25\,\text{kg} \cdot 9.81\,\frac{\text{m}}{\text{s}^2} = 245\,\text{N}$$





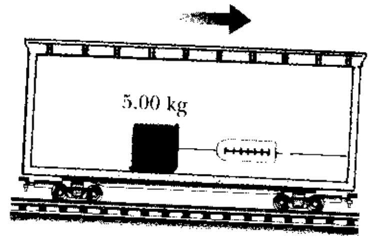
$$\sum F_r = m a_c$$

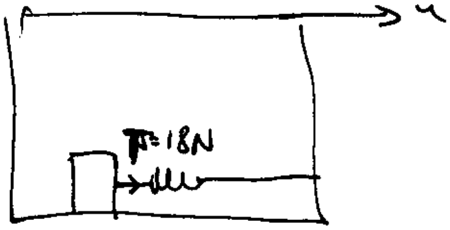
$$T = m a_c = m \frac{v^2}{r} \qquad v = \sqrt{\frac{rT}{m}} \qquad V_{max} = \sqrt{\frac{r T_c}{m}}$$

$$V_{max} = \sqrt{\frac{0.8\,\text{m} \cdot 245\,\text{N}}{3\,\text{kg}}} = 8.08\,\text{m/s}$$

$$\boxed{0 \le V \le 8.08\,\text{m/s}}$$ Range of speeds

● An object of mass 5.00 kg, attached to a spring scale, rests on a frictionless, horizontal surface as shown in Figure P6.19. The spring scale, attached to the front end of a boxcar, has a constant reading of 18.0 N when the car is in motion. (a) The spring scale reads zero when the car is at rest. Determine the acceleration of the car. (b) What constant reading will the spring scale show if the car moves with constant velocity? (c) Describe the forces on the object as observed by someone in the car and by someone at rest outside the car.





inertial) $a=0$

non-inertial) a

$T = ma$

$T - F_f = 0$

$F_f = -ma$

$a = \frac{T}{m} = \frac{18\ \text{N}}{5\ \text{kg}} = 3.60\ \text{m/s}^2$

A small, spherical bead of mass 3.00 g is released from rest at $t = 0$ in a bottle of liquid shampoo. The terminal speed is observed to be $v_T = 2.00$ cm/s. Find (a) the value of the constant b in Equation 6.2, (b) the time t at which the bead reaches $0.632v_T$, and (c) the value of the resistive force when the bead reaches terminal speed.

$$mg - bv = ma$$

Terminal velocity: $(a = 0)$

$$bv_T = mg \rightarrow v_T = \frac{mg}{b} \rightarrow b = \frac{mg}{v_T} = \frac{3\times10^{-3}\,\text{kg} \cdot 9.81\,\frac{m}{s^2}}{0.02\,m/s} = 1.47\ N \cdot \frac{s}{m}$$

a) $\boxed{b = 1.47\ N\,\frac{s}{m}}$

b) $t\,(v = 0.632\,v_T)$

$$v = v_T\left(1 - e^{-bt/m}\right)$$

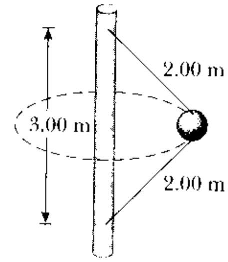
$$0.632\,\cancel{v_T} = \cancel{v_T}\left(1 - e^{-bt/m}\right)$$

$$0.368 = e^{-bt/m} \longrightarrow \ln(0.368) = -\frac{bt}{m} \rightarrow$$

$$\boxed{t = -\frac{m}{b}\ln(0.368) = 2.04\times10^{-3}\,s}$$

c) $\boxed{R = mg = 2.94\times10^{-2}\,N}$

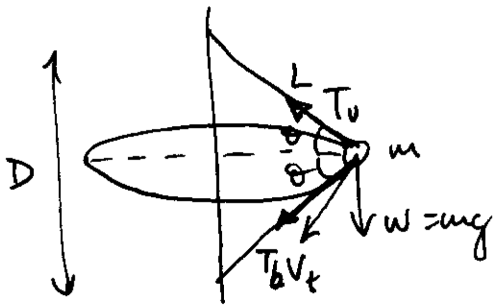
A 4.00-kg object is attached to a vertical rod by two strings as shown in Figure P6.11. The object rotates in a horizontal circle at constant speed 6.00 m/s. Find the tension in (a) the upper string and (b) the lower string.



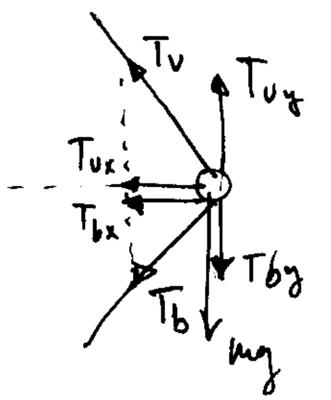
$m = 4\ kg$

$v_t = 6\ m/s$ = constant $(a_t = 0)$

$a_c = \frac{v_t^2}{r}$



$$x)\ F_c = \overbrace{m\frac{v_t^2}{r}}^{m a_c} = T_{Ux} + T_{bx}$$



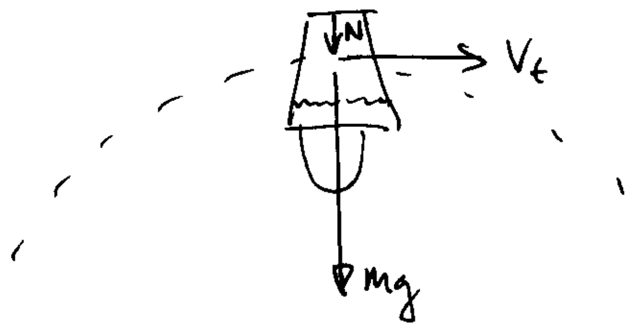
$$y)\ T_{Uy} - T_{by} - mg = 0$$

$$x)\ m\frac{v_t^2}{r} = T_U\cos\theta + T_b\cos\theta = (T_U + T_b)\cos\theta$$

$$y)\ (T_U - T_b)\sin\theta = mg$$

$$\sin\theta = \frac{D/2}{L} \rightarrow \theta \rightarrow \text{solve}$$

▲ A pail of water is rotated in a vertical circle of radius 1.00 m. What is the pail's minimum speed at the top of the circle if no water is to spill out?

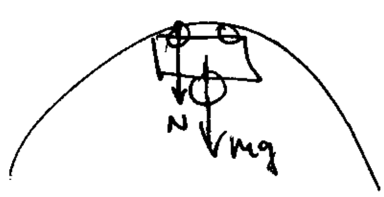


$$\sum F_y = m\frac{V^2}{r} = mg + N \quad \text{(for the water)}$$

At the minimum speed, the normal is zero:

$$\cancel{m}g = \cancel{m}\frac{V_{min}^2}{r} \rightarrow \boxed{V_{min} = \sqrt{rg} = \sqrt{1m \cdot 9.81\frac{m}{s^2}} = 3.13\frac{m}{s}}$$

Problem 18:



a) $a_c = \frac{V^2}{r} \Rightarrow \frac{(13\ m/s)^2}{2 \cdot 9.81\ \frac{m}{s^2}} = 8.62\ m = r$

b) $mg + N = m\frac{V^2}{r}$

$$N = m\left(\frac{V^2}{r} - g\right) = m(\underbrace{2g}_{} - g) = mg$$

c) $r = 20\ m$, $V = 13\ m/s$ $\Big\}$ $a_c = \frac{V^2}{r} = \frac{(13\ m/s)^2}{20\ m} = 8.45\ \frac{m}{s^2}$

$N = m(a_c - g) < 0$ N needs to point out of the center.