

LINEAR MOMENTUM AND COLLISIONS

CONSERVATION OF LINEAR MOMENTUM

Let's assume the collision of two objects (m_1, m_2), under the view of Newton's third law;

$$\vec{F}_{12} = -\vec{F}_{21} \rightarrow \vec{F}_{12} + \vec{F}_{21} = 0$$

We know from Newton's second law that;

$$m_1 \vec{a}_1 + m_2 \vec{a}_2 = 0$$

which is;

$$m_1 \frac{d\vec{v}_1}{dt} + m_2 \frac{d\vec{v}_2}{dt} = 0$$

or;

$$\frac{d}{dt} (m_1 \vec{v}_1 + m_2 \vec{v}_2) = 0$$

Therefore; $m_1 \vec{v}_1 + m_2 \vec{v}_2 = ct$

As we will see, in the absence of external forces, the linear momentum of a system is conserved!

$$\boxed{\vec{p} = m\vec{v}}$$

linear momentum.

Total (net) linear momentum:

$$\vec{P}_T \equiv \vec{P}_N = \sum_i \vec{P}_i = \sum_i m_i \vec{v}_i$$

$$\left. \begin{aligned} P_{Tx} &= \sum_i P_{ix} \\ P_{Ty} &= \sum_i P_{iy} \end{aligned} \right\} \begin{aligned} \text{where } P_x &= m v_x \\ P_y &= m v_y \end{aligned}$$

Newton's second law:

$$\sum \vec{F} = m \vec{a} = m \frac{d\vec{v}}{dt} = \frac{d}{dt}(m\vec{v}) = \frac{d\vec{p}}{dt}$$

$$\sum \vec{F} = \frac{d\vec{p}}{dt}$$

Newton formulated it like that.

This is more general, since it allows to study the case in which mass changes with time.

Conservation of linear momentum:

$$\sum \vec{F} = \underbrace{\sum \vec{F}_{int}}_0 + \sum \vec{F}_{ext} = \frac{d\vec{p}_T}{dt}$$

" Newton's 3rd

Therefore; if $\sum \vec{F}_{ext} = 0$ then;

$$\vec{p}_T = \text{constant}$$

IMPULSE AND MOMENTUM

As seen before, when a net force changes with time, there will be a change in linear momentum $\Delta \vec{p}$. This change is called 'Impulse' (I);

$$\vec{I} = \Delta \vec{p} = \vec{p}_f - \vec{p}_i = \int_{t_i}^{t_f} \vec{F}_N dt = \int_{t_i}^{t_f} \sum \vec{F}_i dt$$

When working with average force ($\sum \vec{F}_{avg}$), then

$$\vec{I} = \Delta \vec{p} = \left(\sum \vec{F} \right)_{avg} \Delta t$$

where $\Delta t = t_f - t_i$

COLLISIONS IN ONE DIMENSION

Elastic collisions: $\Delta K = 0$ $\Delta \vec{p} = 0$

Inelastic collisions: $\Delta K \neq 0$ $\Delta \vec{p} = 0$

Perfectly inelastic collisions (When the objects stick together)

$$v_{1f} = v_{2f} \equiv v$$

Let's see 1-D collisions: ($\vec{v} \rightarrow v$)

⊕ PERFECTLY INELASTIC COLLISIONS (SIMPLEST CASE)

$$\Delta \vec{p}, \Delta K \neq 0 \quad v_{1f} = v_{2f} = v_f$$



$$P_i = P_f \rightarrow m_1 v_{1i} + m_2 v_{2i} = (m_1 + m_2) v_f$$

$$v_f = \frac{m_1 v_{1i} + m_2 v_{2i}}{m_1 + m_2}$$

⊕ ELASTIC COLLISIONS :

$$\Delta p = 0, \Delta K = 0$$

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$$

$$\frac{1}{2} m_1 v_{1i}^2 + \frac{1}{2} m_2 v_{2i}^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2$$

complicated situation.

$$v_{1f} = \left(\frac{m_1 - m_2}{m_1 + m_2} \right) v_{1i} + \left(\frac{2m_2}{m_1 + m_2} \right) v_{2i}$$

$$v_{2f} = \left(\frac{2m_1}{m_1 + m_2} \right) v_{1i} + \left(\frac{m_2 - m_1}{m_1 + m_2} \right) v_{2i}$$

some special situations:

$$(A) \quad m_1 = m_2 \rightarrow v_{1f} = v_{2i} \text{ and } v_{2f} = v_{1i} \quad (\text{billiard})$$

$$(B) \quad v_{2i} = 0 \rightarrow v_{1f} = \left(\frac{m_1 - m_2}{m_1 + m_2} \right) v_{1i}$$

$$v_{2f} = \left(\frac{2m_1}{m_1 + m_2} \right) v_{1i}$$

$$\rightarrow \text{and } m_1 = m_2 : \left| \begin{array}{l} v_{1f} = 0 \\ v_{2f} = v_{1i} \end{array} \right. \quad (\text{hanging balls})$$

$$(C) \quad m_1 \gg m_2 \rightarrow \begin{array}{l} v_{1f} \approx v_{1i} \\ \text{and } v_{2i} = 0 \\ v_{2f} \approx v_{1i} \end{array}$$

$$m_1 \ll m_2 \rightarrow \begin{array}{l} v_{1f} \approx -v_{1i} \\ \text{and } v_{2i} = 0 \\ v_{2f} \approx 0 \end{array}$$

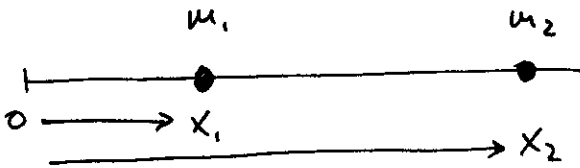
COLLISIONS IN TWO DIMENSIONS

Elastic: $\Delta \vec{p} = 0, \Delta K = 0$ $\Delta p_x = 0, \Delta p_y = 0$

Inelastic: $\Delta \vec{p} = 0, \Delta K \neq 0$

THE CENTER OF MASS

a) Let's consider two particles:



the center of mass is at
$$x_{CM} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$

It is like having a mass ($M = m_1 + m_2$) at a position (x_{CM})

b) For a system of many particles:

$$x_{CM} = \frac{1}{M} \sum_i m_i x_i \quad \text{where} \quad M = \sum_i m_i$$

c) In more than one dimension:

$$y_{CM} = \frac{1}{M} \sum_i m_i y_i \quad \text{and} \quad z_{CM} = \frac{1}{M} \sum_i m_i z_i$$

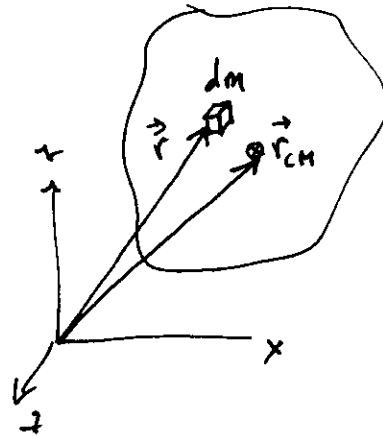
d) in general :

$$\vec{r}_{cm} = x_{cm} \hat{i} + y_{cm} \hat{j} + z_{cm} \hat{k}$$

$$\vec{r}_{cm} = \frac{1}{M} \sum_i m_i \vec{r}_i$$

c) Center of mass of a continuous object :

$$\vec{r}_{cm} = \frac{1}{M} \int \vec{r} dm$$



9.6. MOTION OF A SYSTEM OF PARTICLES

We could analyze the motion of a system of particles by looking at its center of mass.

$$\vec{r}_{cm} = \frac{1}{M} \sum_i m_i \vec{r}_i$$

$$\text{Velocity: } \vec{v}_{cm} = \frac{d\vec{r}_{cm}}{dt} = \frac{1}{M} \sum_i m_i \vec{v}_i$$

$$\text{Linear momentum: } \vec{P} = M \vec{v}_{cm} = \sum_i m_i \vec{v}_i$$

Acceleration : $\vec{a}_{cm} = \frac{d\vec{v}_{cm}}{dt} = \frac{1}{M} \sum_i m_i \vec{a}_i$

Newton's second law : $M\vec{a}_{cm} = \sum_i m_i \vec{a}_i = \sum_i \vec{F}_i$

(where \vec{F}_i is the net force on particle i)

As we have seen before :

$$\sum_i \vec{F}_i = \sum_i \vec{F}_{int} + \sum_i \vec{F}_{ext}$$

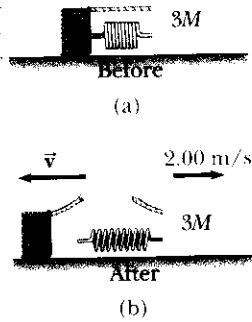
therefore, the acceleration of the center of mass of a system is only determined by external forces.

$$M\vec{a}_{cm} = \sum_i \vec{F}_{ext}$$

[We won't see sections 9.7 and 9.8]

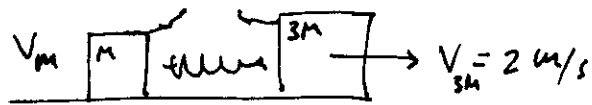
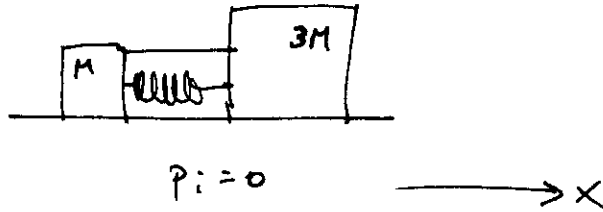
● Two blocks of masses M and $3M$ are placed on a horizontal, frictionless surface. A light spring is attached to one of them, and the blocks are pushed together with the spring between them (Fig. P9.4). A cord initially holding the blocks together is burned; after that happens, the block of mass $3M$ moves to the right with a speed of 2.00 m/s . (a) What is the velocity of the block of mass M ?

(b) Find the system's original elastic potential energy, taking $M = 0.350 \text{ kg}$. (c) Is the original energy in the spring or in the cord? Explain your answer. (d) Is momentum of the system conserved in the bursting-apart process? How can it be, with large forces acting? How can it be, with no motion beforehand and plenty of motion afterward?



$$(a) \Delta p = 0$$

$$p_i = p_f$$



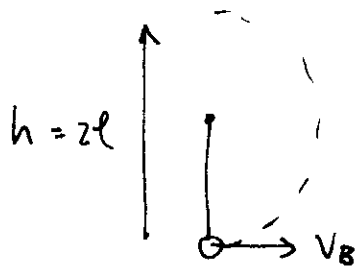
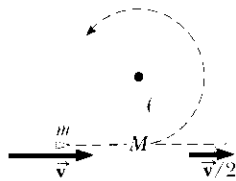
$$p_f = M v_M + 3M v_{3M}$$

$$v_M = -\frac{3M v_{3M}}{M} = -3 v_{3M} = -6 \text{ m/s}$$

$$(b) \text{ Energy } E_i = \frac{1}{2} k x^2 = E_f = \frac{1}{2} M v_M^2 + \frac{3}{2} M v_{3M}^2 = 8.40 \text{ J}$$

Energy was stored by the spring.

As shown in Figure P9.18 (page 262), a bullet of mass m and speed v passes completely through a pendulum bob of mass M . The bullet emerges with a speed of $v/2$. The pendulum bob is suspended by a stiff rod of length ℓ and negligible mass. What is the minimum value of v such that the pendulum bob will barely swing through a complete vertical circle?



$$\frac{1}{2} M v_B^2 = M g 2\ell$$

$$\boxed{v_B^* = \sqrt{2g2\ell} = \sqrt{4g\ell} = 2\sqrt{g\ell}}$$

(before)

 m_b
 \Rightarrow
 v_{bi}
 m_B
 \circ
 $v_{B:} = 0$

(after)

 m_b
 \circ
 v_{bf}
 m_B
 \circ
 v_{Bf}

$$\Delta p = 0 \quad p_f = p_i = 0$$

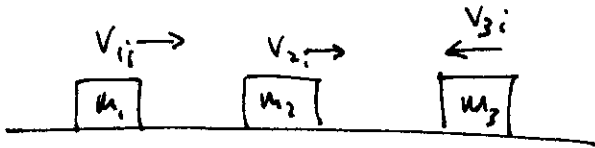
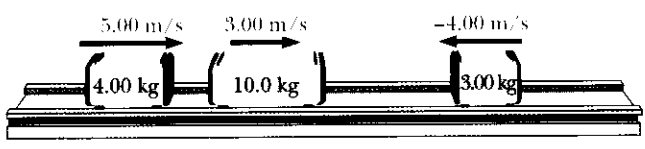
$$m_b v_{bf} + m_B v_{Bf} = m_b v_{bi} = 0$$

$$v_{Bf} = v_B^* = 2\sqrt{g\ell}$$

$$v_{bf} = \frac{v_{bi}}{2}$$

$$\boxed{v_{bi} = \frac{4m_B}{m_b} \sqrt{g\ell}}$$

● (a) Three carts of masses 4.00 kg, 10.0 kg, and 3.00 kg move on a frictionless, horizontal track with speeds of 5.00 m/s, 3.00 m/s, and 4.00 m/s as shown in Figure P9.24. Velcro couplers make the carts stick together after colliding. Find the final velocity of the train of three carts. (b) **What If?** Does your answer require that all the carts collide and stick together at the same moment? What if they collide in a different order?



$$\begin{aligned}
 m_1 &= 4 \text{ kg} \\
 v_{1i} &= 5 \text{ m/s} \\
 m_2 &= 10 \text{ kg} \\
 v_{2i} &= 3 \text{ m/s} \\
 m_3 &= 3 \text{ kg} \\
 v_{3i} &= -4 \text{ m/s}
 \end{aligned}$$

a) $\Delta P = 0 \quad P_f = P_i$

$$P_f = (m_1 + m_2 + m_3) V_f = P_i = m_1 v_1 + m_2 v_2 + m_3 v_3$$

$$\boxed{V_f = \frac{m_1 v_1 + m_2 v_2 + m_3 v_3}{m_1 + m_2 + m_3} = +2.24 \text{ m/s}}$$

(b) No, they do not need to collide at the same time.

Let's assume m_2 and m_3 collide first.

$$P_f = (m_2 + m_3) V_{23} = m_2 v_2 + m_3 v_3 = P_i$$

$$V_{23} = +1.38 \text{ m/s}$$

Then $(m_2 + m_3)$ collide with m_1

$$P_f = (m_1 + m_2 + m_3) V_{123} = (m_2 + m_3) V_{23} + m_1 v_1 = P_i$$

$$\boxed{V_{123} = +2.24 \frac{\text{m}}{\text{s}} = V_f}$$

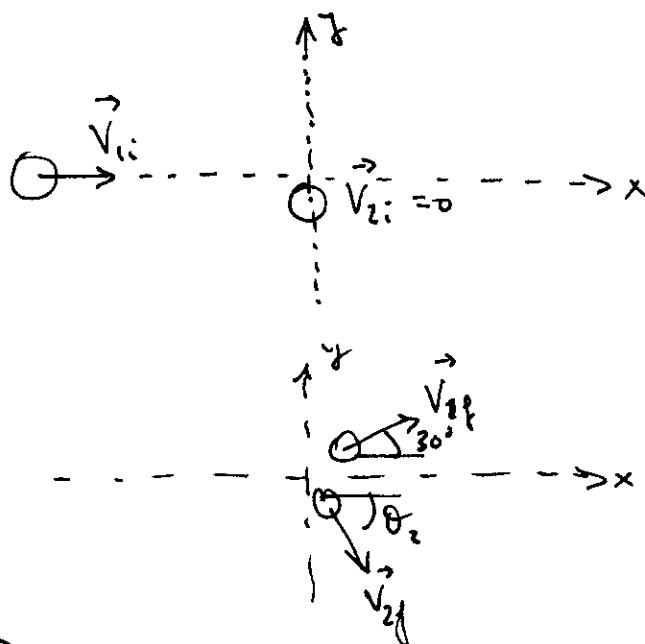
A billiard ball moving at 5.00 m/s strikes a stationary ball of the same mass. After the collision, the first ball moves at 4.33 m/s, at an angle of 30.0° with respect to the original line of motion. Assuming an elastic collision (and ignoring friction and rotational motion), find the struck ball's velocity after the collision.

$$\vec{V}_{1i} = V_{1ix} = 5.00 \text{ m/s}$$

$$\vec{V}_{2i} = 0$$

$$V_{1f} = 4.33 \text{ m/s at } \theta_1 = 30^\circ$$

$$\boxed{\vec{V}_{2f} ?}$$



$$\Delta \vec{P} = 0$$

$$\Delta P_x = 0 \quad \Delta P_y = 0$$

↓

$$x) P_{fx} - P_{ix} = m_1 V_{1f} \cos 30^\circ - m_2 V_{2fx} = 0 \quad \boxed{V_{2fx} = 1.25 \text{ m/s}}$$

$$y) P_{fy} - P_{iy} = m_1 V_{1f} \sin 30^\circ - (-m_2 V_{2fy}) = 0 \quad \boxed{V_{2fy} = -2.16 \text{ m/s}}$$

$$\boxed{V_{2f} = \sqrt{V_{2fx}^2 + V_{2fy}^2} = 2.50 \text{ m/s}}$$

$$\boxed{\theta_2 = \tan^{-1} \left(\frac{V_{2fy}}{V_{2fx}} \right) = -60^\circ}$$

▲ A 12.0-g wad of sticky clay is hurled horizontally at a 100-g wooden block initially at rest on a horizontal surface. The clay sticks to the block. After impact, the block slides 7.50 m before coming to rest. If the coefficient of friction between the block and the surface is 0.650, what was the speed of the clay immediately before impact?

$$m = 12 \text{ g}$$

$$M = 100 \text{ g}$$

$$d = 7.50 \text{ m}$$

$$\mu_k = 0.65$$

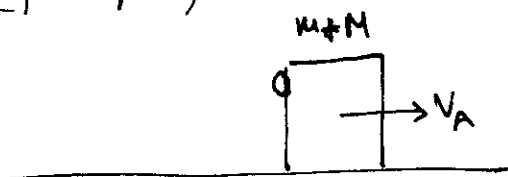
$$v = ?$$

(before)

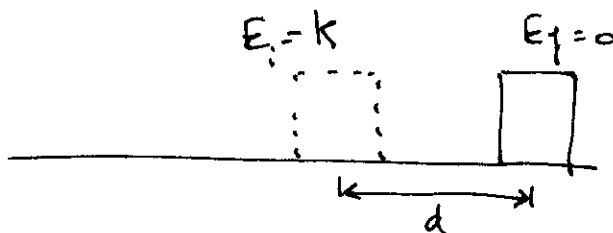


$$P_{\text{before}} = mv$$

(just after)



$$P_{\text{after}} = (m+M)v_A$$

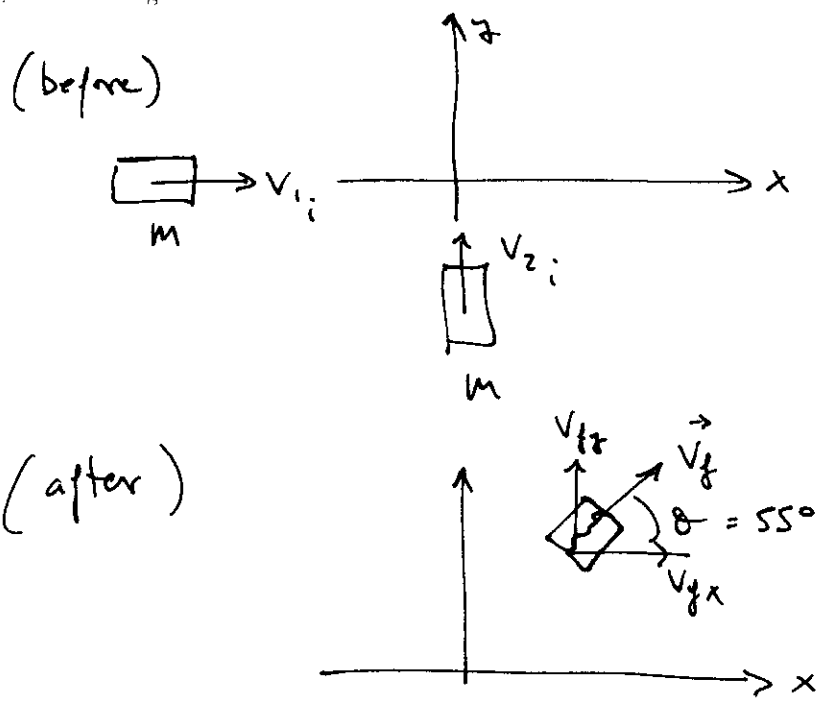


$$\Delta E = E_f - E_i = -\frac{1}{2}(m+M)v_A^2 = W_R = -(m+M)gd$$

$$v_A = 9.77 \text{ m/s}$$

$$v = \frac{m+M}{m} v_A = 91.2 \frac{\text{m}}{\text{s}}$$

● Two automobiles of equal mass approach an intersection. One vehicle is traveling with velocity 13.0 m/s toward the east, and the other is traveling north with speed v_2 . Neither driver sees the other. The vehicles collide in the intersection and stick together, leaving parallel skid marks at an angle of 55.0° north of east. The speed limit for both roads is 35 mi/h, and the driver of the northward-moving vehicle claims he was within the speed limit when the collision occurred. Is he telling the truth? Explain your reasoning.



$\theta = 55^\circ$
 $v_1 = 13 \text{ m/s}$
 ~~$v_2 = 35 \text{ mi/h}$~~
 $v_2 < 35 \text{ mi/h} ?$

$$\begin{aligned} \Delta P_x = 0 &\rightarrow P_{xi} = P_{xf} \rightarrow \cancel{m} v_1 = 2m v_{fx} = 2\cancel{m} v_f \cos \theta \\ \Delta P_y = 0 &\rightarrow P_{yi} = P_{yf} \rightarrow \cancel{m} v_2 = 2m v_{fy} = 2\cancel{m} v_f \sin \theta \end{aligned} \quad \Bigg\}$$

$$v_1 = 2 v_f \cos \theta$$

$$v_2 = 2 v_f \sin \theta$$

$$\frac{v_1}{v_2} = \frac{\cos \theta}{\sin \theta}$$

$$v_2 = v_1 \tan \theta = 13 \frac{\text{m}}{\text{s}} \tan 55^\circ = 18.6 \frac{\text{m}}{\text{s}}$$

$(v_2 = 41.5 \text{ mi/h} \gg \text{speed limit})$