STATIC EQUILIBRIUM AND ELASTICITY

THE RIGID OBJECT IN EQUILIBRIUM

The conditions to have a "rigid" object in equilibrium are:

1) $\sum \vec{F} = 0 \quad (m\vec{a} = 0) \qquad \sum F_i = 0 \quad (i = x, y, z)$

2) $\sum \vec{\tau} = 0 \quad (I\vec{\alpha} = 0)$

ELASTIC PROPERTIES OF SOLIDS

stress : external force per unit cross sectional area

strain : measure of the degree of deformation and proportional to the stress

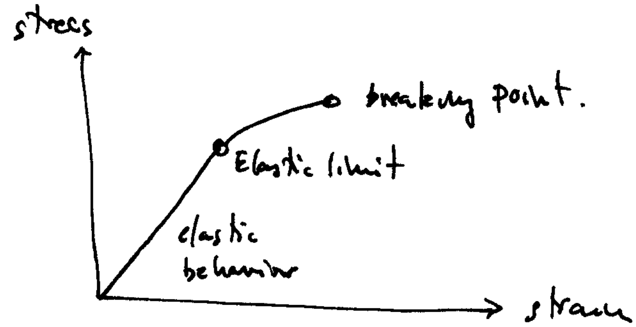
$$\text{Elastic modulus} \equiv \frac{\text{stress}}{\text{strain}} \quad (\sim \text{spring constant})$$

Young's modulus : resistance of a solid to a change in length

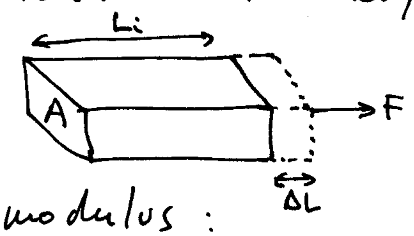
Shear modulus: resistance to motion of the planes within a solid parallel to each other

Bulk modulus: resistance of a solid to a change in volume.

Young's modulus:

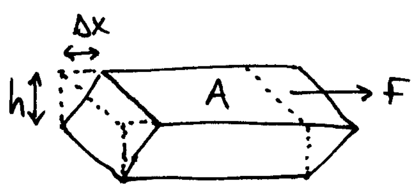


$$Y = \frac{\text{tensile stress}}{\text{tensile strain}} = \frac{F/A}{\Delta L / L_i}$$



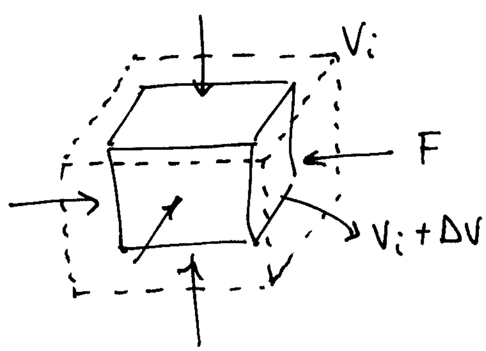
Shear modulus:

$$S = \frac{\text{shear stress}}{\text{shear strain}} = \frac{F/A}{\Delta x / h}$$

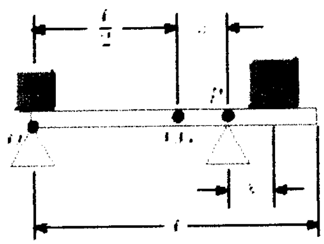


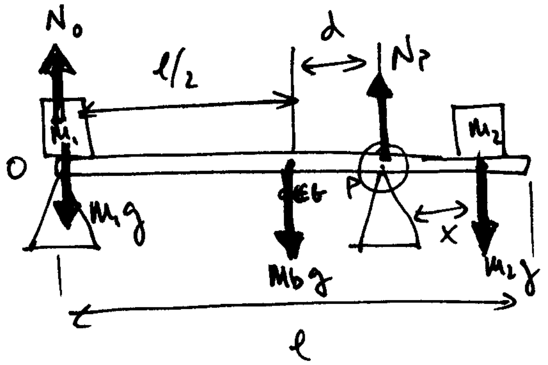
Bulk modulus:

$$B = \frac{\text{volume stress}}{\text{volume strain}} = -\frac{\Delta F / A}{\Delta V / V_i} = -\frac{\Delta P}{\Delta V / V_i}$$



A uniform beam of mass m_b and length l supports blocks with masses m_1 and m_2 at two positions. The beam rests on two knife edges. For what value of x will the beam be balanced at P such that the normal force at O is zero? (Use m_b for m_b, m_1 for m_1, m_2 for m_2, L for l, and d as appropriate in your equation.





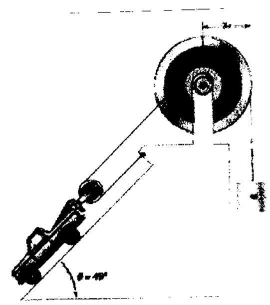
Let's take torques about P (which is the possible rotation point)

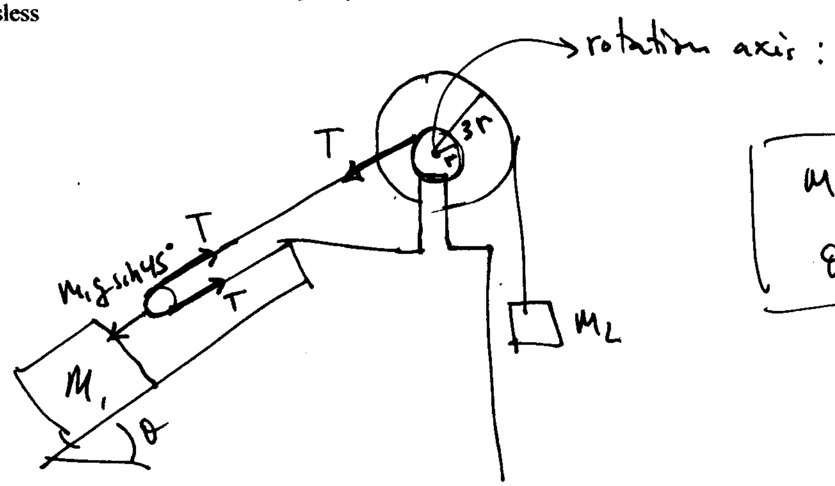
$$\sum \tau_P = -N_o\left[\frac{l}{2}+d\right] + m_1 g\left[\frac{l}{2}+d\right] + m_b g d - m_2 g x = 0$$

we want to find x for which $N_o = 0$;

$$\boxed{x = \frac{(m_1 g + m_b g)d + m_1 g \frac{l}{2}}{m_2 g} = \frac{(m_1 + m_b)d + m_1 \frac{l}{2}}{m_2}}$$

Find the mass m of the counterweight needed to balance the 1500 kg truck on the incline. Assume all pulleys are frictionless and massless





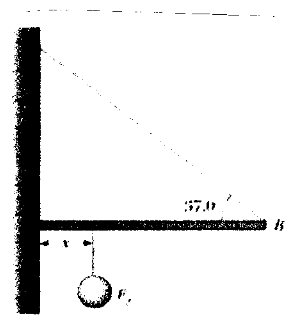
$$\boxed{\begin{array}{l} m_1 = 1500\text{ kg} \\ \theta = 45° \end{array}}$$

$$\left.\begin{array}{l} \sum \tau = -mg(3r) + Tr = 0 \\ 2T = m_1 g \sin\theta \end{array}\right\}$$

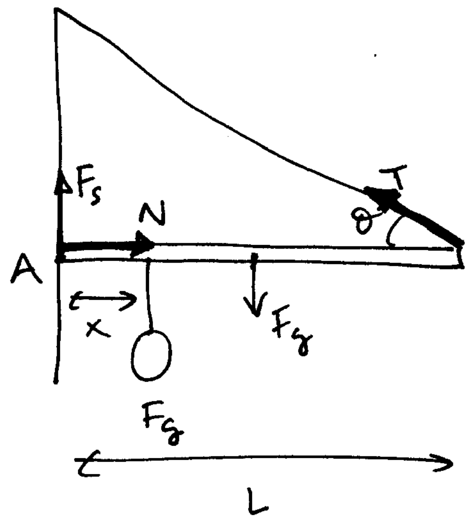
$$m = \frac{Tr}{3gr} = \frac{m_1 \cancel{g} \sin\theta \cancel{r}}{6 \cancel{g} \cancel{r}} = \frac{m_1 \sin\theta}{6}$$

$$\boxed{m = 177\text{ kg}}$$

One end of a uniform 4 m long rod of weight F_g is supported by a cable. The other end rests against the wall, where it is held by friction. The coefficient of static friction between the wall and the rod is $\mu_s = 0.5$. Determine the minimum distance x from point A at which an additional weight F_g (the same as the weight of the rod) can be hung without causing the rod to slip at point A.



$$\boxed{\begin{aligned}\theta &= 37^\circ \\ \mu_s &= 0.5 \\ L &= 4\,m\end{aligned}}$$



$\underline{F_s = \mu_s N}$ just before slipping

1) $\sum F_x = 0 \longrightarrow N - T\cos\theta = 0 \rightarrow \boxed{N = T\cos\theta}$

$$\boxed{F_s = \mu_s T\cos\theta}$$

2) $\sum F_y = 0 \longrightarrow F_s - F_g - F_g + T\sin\theta = 0$

$$\mu_s T\cos\theta - 2F_g + T\sin\theta = 0$$

$$T(\sin\theta + \mu_s\cos\theta) = 2F_g \qquad \boxed{T = \frac{2F_g}{\sin\theta + \mu_s\cos\theta}}$$

3) $\sum \tau_A = 0 \qquad -F_g x - F_g\frac{L}{2} + LT\sin\theta = 0$

$$x = \frac{-F_g\frac{L}{2} + T\sin\theta L}{F_g} = \frac{-\cancel{F_g}\frac{L}{2} + \frac{2\cancel{F_g}\sin\theta}{\sin\theta + \mu_s\cos\theta}L}{\cancel{F_g}}$$

$$\boxed{x = \left[\frac{2\sin\theta}{\sin\theta + \mu_s\cos\theta} - \frac{1}{2}\right]L = 2.82\,m}$$

Assume Young's modulus for bone is 1.50×10^{10} N/m^2. The bone breaks if stress greater than 1.50×10^8 N/m^2 is imposed on it. (a) What is the maximum force that can be exerted on the femur bone in the leg if it has a minimum effective diameter of 2.50 cm? (b) If this much force is applied compressively, by how much does the 25.0-cm-long bone shorten?

a) $$\text{stress} = \frac{F}{A} = \frac{F}{\pi r^2} \Rightarrow \boxed{F = (\text{stress})\,\pi\left(\frac{d}{2}\right)^2 = 73.6\text{ kN}}$$

b) $$\text{stress} = Y(\text{strain}) = \frac{Y\,\Delta L}{L_i}$$

$$\boxed{\Delta L = \frac{(\text{stress})\,L_i}{Y} = \frac{(1.5\times10^8\text{ N/m}^2)(0.250\text{ m})}{1.50\times10^{10}\text{ N/m}^2} = 2.50\text{ mm}}$$

A child slides across a floor in a pair of rubber-soled shoes. The friction force acting on each foot is 20.0 N. The footprint area of each shoe sole is 14.0 cm^2, and the thickness of each sole is 5.00 mm. Find the horizontal distance by which the upper and lower surfaces of each sole are offset. The shear modulus of the rubber is 3.00 MN/m^2.

From the definition of shear modulus $S = \frac{\text{shear stress}}{\text{shear strain}} = \frac{F/A}{\Delta x / h}$

$$\Delta x = \frac{hF}{SA} = \frac{(5\times10^{-3}\,m)(20\,N)}{(3\times10^{6}\,N/m^2)(14\times10^{-4}\,m^2)} = 2.38\times10^{-5}\,m$$